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AN INVESTIGATION OF FLUX DENSITY DETERMINATIONS.(U)

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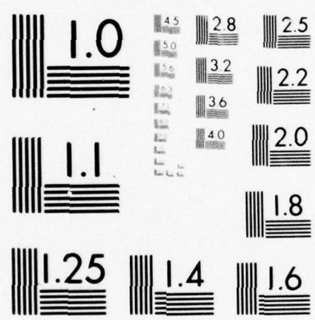

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AN INVESTIGATION OF FLUX DENSITY DETERMINATIONS

SOUTHWEST RESEARCH INSTITUTE
SAN ANTONIO, TEXAS 78284

DECEMBER 1976

FINAL REPORT FOR PERIOD JULY 1976 - DECEMBER 1976

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AIR FORCE MATERIALS LABORATORY
AIR FORCE WRIGHT AERONAUTICAL LABORATORIES
AIR FORCE SYSTEMS COMMAND
WRIGHT-PATTERSON AIR FORCE BASE, OHIO 45433

NOTICE

This final report was submitted by Southwest Research Institute, San Antonio, Texas 78284, under contract No. F33615-76-C-5305, with the Air Force Materials Laboratory, Air Force Wright Aeronautical Laboratories, AFSC, Wright-Patterson Air Force Base, Ohio 45433. James A. Holloway, AFML/LLP, was the Air Force Materials Laboratory Project Monitor.

This report has been reviewed by the Information Office (IO) and is releasable to the National Technical Information Service (NTIS). At NTIS, it will be available to the general public, including foreign nations.

This technical report has been reviewed and is approved for publication.

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Project Monitor

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AIR FORCE - 26 APRIL 77 - 500

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REPORT DOCUMENTATION PAGE		READ INSTRUCTIONS BEFORE COMPLETING FORM
1. REPORT NUMBER: AFML-TR-76-236	2. GOVT ACCESSION NO.	3. RECIPIENT'S CATALOG NUMBER
4. TITLE (and Subtitle): AN INVESTIGATION OF FLUX DENSITY DETERMINATIONS,		5. TYPE OF REPORT & PERIOD COVERED: Final rept., 1 Jul[redacted]76—31 Dec[redacted] 76
		6. PERFORMING ORG. REPORT NUMBER: SwRI-15-4618
7. AUTHOR(s): Robert E./Beissner		8. CONTRACT OR GRANT NUMBER(s): F33615-76-C-5305 new
9. PERFORMING ORGANIZATION NAME AND ADDRESS: Southwest Research Institute, 8500 Culebra Road, San Antonio, Texas 78284		10. PROGRAM ELEMENT, PROJECT, TASK AREA & WORK UNIT NUMBERS: DOD Element 62102F, 73510954 Ø9
11. CONTROLLING OFFICE NAME AND ADDRESS: Air Force Materials Laboratory (AFML/LLP), Air Force Wright Aeronautical Laboratories, AFSC, Wright-Patterson Air Force Base, OH 45433		12. REPORT DATE: Dec[redacted]76
		13. NUMBER OF PAGES: 71 79p.
14. MONITORING AGENCY NAME & ADDRESS (if different from Controlling Office)		15. SECURITY CLASS. (of this report): Unclassified
		15a. DECLASSIFICATION/DOWNGRADING SCHEDULE

16. DISTRIBUTION STATEMENT (of this Report)

Approved for public release; distribution unlimited

17. DISTRIBUTION STATEMENT (of the abstract entered in Block 20, if different from Report)

18. SUPPLEMENTARY NOTES

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

magnetic particle inspection
magnetic particles
nondestructive testing
magnetic flux leakage
magnetic flux calculations
magnetic field measurement

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Two unresolved problems associated with magnetic particle inspection are analyzed. These are the determination of flux density requirements and the measurement of flux densities in specimens to be inspected. It is concluded that the development of a mathematical model for the prediction of flux density requirements will require the determination of detectability criteria, the development of a theory of time-dependent diffusion of particles in inhomogeneous fields, and the development of methods for predicting flux leakage distributions associated → next page

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with flaws. Also, it is suggested that Hall probe measurements of tangential field strengths at the surface of a specimen are probably adequate as a pre-inspection step to insure proper magnetization. A recommended program of research leading to the development and experimental verification of a mathematical model, and the development of appropriate flux density instrumentation, is presented.

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I. INTRODUCTION AND SUMMARY

This report contains the results of a study performed under Contract No. F33615-76-C-5305 pertaining to magnetic particle inspection specifications. The objective of the work was to plan a program of research to develop (1) methods for predicting flux density requirements in inspections of simple and complex parts, and (2) instrumentation for measuring flux densities. It is expected that the results of the follow-on program will be incorporated in improved specifications for magnetic particle inspection.

The principal problem of concern here is that of determining ranges of values of magnetic flux densities within which magnetic particle flaw detection is assured, as a function of the many variables encountered in practical inspection problems. As is noted in the text of this report, there is ample evidence such magnetization requirements are poorly understood at present, and that this often leads to a failure to detect flaws that should not be missed.

There are two reasons why this situation exists, the first being that the proper level of magnetization appears to be a complicated function of several test parameters. It is well known, for example, that the gross geometry of the specimen plays an important role, as does the permeability of the material and the method of magnetization. Other factors, such as the type of flaw, its geometrical shape, location, and the possible existence of associated strain fields are also thought to influence flux density requirements.

The second reason that the magnetization problem still exists, after several years of general use of the magnetic particle method, is that there has not yet been a serious attempt to analyze all of the factors that contribute to the formation of an indication. Several experimental studies have been conducted in which one or more parameters have been varied, and these have been useful in identifying certain important factors, such as the mobility of particles in suspension. In recent years there have also been a few attempts to use theoretical models to study parts of the problem, for example, the influence of flaw geometry on the flaw leakage field. Still, there has been no coherent effort to assess the relative significance of all such factors, and to use this information to develop quantitative guidelines for the inspection process.

In the present program one of our aims has been to plan a research program leading to a quantitative, predictive model of the inspection process. One of the requirements we have set in structuring such a program is that it include tests of significance, as described above, so that the final model will include only those test parameters that are important, and will exclude factors that do not play a significant role in flaw detection. In this way the model, and the guidelines it will generate, will contain as few parameters as possible, thus simplifying the determination of flux density requirements.

The second objective of our study was to plan a program for the development of a flux density sensor, an instrument that an inspector would use in a pre-inspection measurement to determine that a specimen is properly magnetized. This is also an important aspect of the eventual solution of the magnetization problem because, with the complicated specimen shapes encountered in practice, there is virtually no way, except by actual measurement, to determine flux densities at all critical points on a specimen.

Our approach to the development of a research plan consisted of three tasks. First we used the computerized bibliographic services of the Nondestructive Testing Information Analysis Center (NTIAC) to search the literature on magnetic particle testing and related topics. Once this was done, selected articles were reviewed, and theoretical and experimental methods that might prove useful in the follow-on program were identified. We then undertook a critical examination of such methods and attempted to assess their potential usefulness in realizing the long-range objectives of the follow-on program. Once this analysis was completed we had enough information in hand to structure a recommended sequence of research tasks that would make use of available methods insofar as is possible, developing improvements as needed, in reaching the final objectives.

A summary of our findings regarding available methods, and a list of recommended research tasks is given in Table 1. Here we show, as the four column headings, the major problem areas where more research is needed. The first three, those pertaining to detectability, and particle diffusion and leakage models, comprise the elements of the predictive model. The last column has to do with the development of flux density instrumentation.

In the first row on this chart we list the objectives of each major research task, and in the second row we summarize the results of our literature survey and methods study. The last two rows contain a list of the specific research tasks that comprise our recommended research plan. A full description of each of these tasks, and how they should be sequenced, is given in Section III of this report.

As a general rule, the theoretical program is structured so as to first find answers to the question of accuracy requirements for the predictive model. This is done to provide quantitative criteria for judging the adequacy of existing theoretical models, thus determining where improvements are needed. Actual tests of existing theoretical models occur in the next phase, through comparisons of predictions with experimental results. The experiments we have in mind here are basic, in the sense that they should be designed to test the various assumptions and approximations involved in the theoretical models. The intent is to determine to what extent the simpler theories are useful, and to identify areas where improvement is needed. As this is being done, the relative significance of factors such as flaw geometry, permeability, etc. should become apparent, thus leading to a simplification of the combined predictive model by the elimination of parameters that do not significantly influence the prediction of detectability.

TABLE I. SUMMARY OF UNRESOLVED PROBLEMS AND RECOMMENDED RESEARCH

	Determine Detectability Criteria	Develop Particle Diffusion Model	Develop Flux Leakage Model	Develop Flux Density Instrument
Problem	Determine particle density needed for detectable indication	Predict particle density in a given inhomogeneous field	Predict leakage field for a given unperturbed flux density, B_o	Measure B_o in specimen to be tested
Status	No useful data available	Equilibrium theory available; not applicable to dry method, marginally applicable to wet method; new models of both methods were developed during the present project but not yet tested	Several methods available; inadequate comparisons with experiment	Tangential Hall probe available; data on use of tangential measurement to determine B_o is not available
Analytical Tasks	Investigate use of visibility theory to predict detectability criteria	Calculate particle distributions with wet and dry methods with simulated leakage fields; investigate effect of method of magnetization (coil or current, AC or DC)field strength and gradient	Test simpler models by comparison with experiment; develop corrections as needed Parameters to be varied include flaw geometry, B-H curve, B_o, method of magnetization	Apply flux density model to calculation of near surface field; estimate error in measurement of B_o by tangential Hall probe method
Experimental Tasks	Determine detectability criteria in terms of number of particles or dimensions of indication	Determine shape, size distribution and susceptibility of particles, also diffusion constant in liquid; measure particle densities (or dimensions of indications) in known fields for comparison with calculations	Measure leakage fields for comparison with calculations	Test tangential probe method by measuring H_t and B_o as function of applied field. Tests should include differen B-H curves and both simple and complex geometry

The next phase of the program involves the development of improved models, as needed, and further comparisons with experiment. Once satisfactory agreement between theory and experiment has been achieved, the various elements of the model will be combined and applied to the generation of data on recommended flux densities as a function of those test parameters that are found to be significant.

In the sensor development phase of the program the general approach is similar. Here, however, because an existing technique involving the use of Hall probes to measure tangential fields at the surface may be all that is needed, initial emphasis is placed on testing the existing technique with commercially available sensors.

Although it is thought that the recommended plan defined in Section III is, in the long run, the most efficient way to proceed, the advantages and disadvantages of alternate approaches are also discussed.

Finally, at the end of Section III, we also discuss alternate ways that the recommended plan might be implemented. The most efficient way, in terms of both funding and calendar time requirements, would be by means of a single contract for the entire program. If this is not feasible, one might consider incremental support, the first increment being that part of the program where accuracy requirements are defined and existing methods evaluated by comparison with experiment. Still another approach would be to simply apply existing methods to the generation of interim flux density guidelines, and then proceed with the program as recommended.

However, regardless of how the follow-on program is implemented, we think it is essential that the interplay between theory and experiment, as outlined in the recommended plan, be preserved. Our review of the literature has revealed that there have been several purely empirical studies, fewer theoretical investigations, and no comprehensive investigation involving both theory and experiment. This is, we think, the principal reason that a scientific basis for magnetic particle inspection is still lacking.

II. TECHNICAL DISCUSSION

A. Background

1. Inspection Methods

The magnetic particle inspection method is a versatile technique that can be applied in many ways to the detection of flaws near the surface in ferromagnetic materials. Because several reviews of magnetic particle methodology are readily available in the open literature[1-5], we will, in this Section, give only a brief review of the basic principle and techniques.

At the top of Figure 1 we illustrate what happens in the vicinity of a discontinuity, in this case a crack, in the surface of a magnetized specimen surrounded by air. The existence of free (uncompensated) poles on the surface of the discontinuity gives rise to a leakage of the lines of induction, here represented by arrows, into the air immediately above the flaw. If magnetic particles are present in this inhomogeneous leakage field, they will be attracted toward regions of higher field strength and thus will tend to accumulate around the crack. If the field is strong enough, and if other conditions are favorable (e.g. the particles are free to migrate to the crack), the density of particles in the vicinity of the crack will, in a short time, become much greater than the density in a flaw-free region, thus creating a visible indication of the presence of a flaw.

Unless the material has a very high retentivity and has already been magnetized by some means, it is necessary to apply the particles while the specimen is subjected to a magnetizing field in order to achieve adequate leakage fields. When this is done the test method is said to be continuous; when particles are applied after the magnetizing field has been removed, the method is called residual.

Some of the more commonly used methods of magnetization are also illustrated in Figure 1. On the left side, Figures 1(a) and 1(b), we show two methods of current magnetization, while in Figure 1(c) and 1(d) we illustrate the yoke and coil methods of magnetization. One method not illustrated here employs a central current conductor running through a hollow specimen, such as a tube. The field induced in the specimen in this case is much like that shown in Figure 1(a), i.e. the lines of induction form circular loops around the circumference of the piece. In all cases the magnetizing current may be AC, DC or half-wave rectified.

Although, when inspecting specimens of complex shape, the choice of a method of magnetization is often dictated by what is feasible, it is important that the method used produce, as nearly as possible, a direction of magnetization perpendicular to the major dimension of the flaw for which the inspection is designed. This is because the leakage field is maximized under such conditions and the probability of obtaining a visible indication is then optimized. Thus when inspecting

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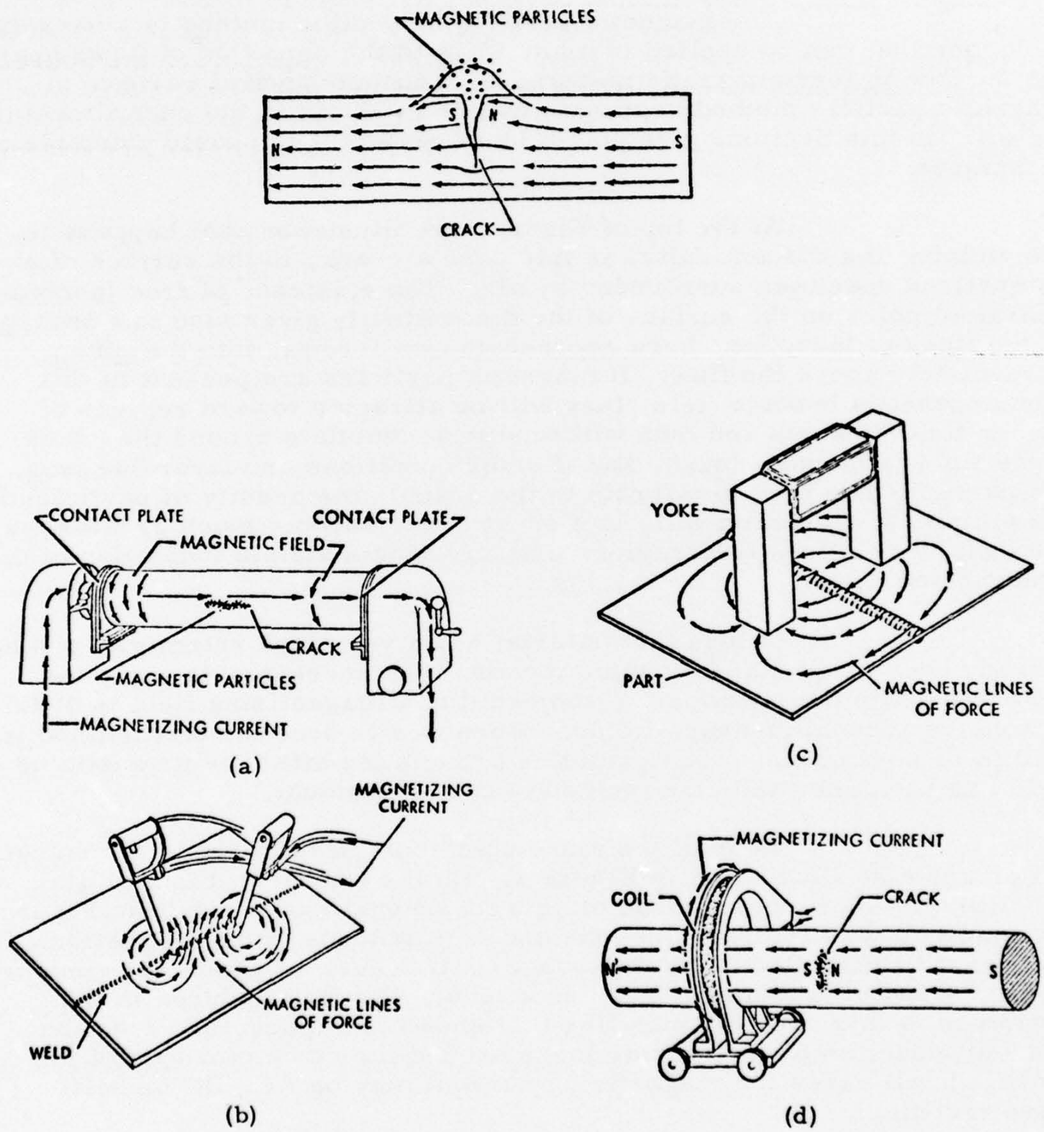

FIGURE 1. MAGNETIC PARTICLE INSPECTION PRINCIPLE AND METHODS OF IMPLEMENTATION (Ref. 1)

for longitudinal cracks in a cylindrical specimen, one would choose the current method, as illustrated in Figure 1(a); for transverse flaws the coil method is better as shown in Figure 1(d).

There are both wet and dry methods of particle application. With the dry method particles are applied in the form of a powder, which is sprinkled on the piece while it is magnetized. Excess powder is then blown off with a light stream of air before the source of magnetization is turned off. With the wet method the particles are suspended in a liquid which is applied to the specimen. When the surface is covered the magnetizing field is applied, and particles are drawn out of suspension and tend to accumulate at discontinuities.

The types of particles one may use comprise another variation of the basic technique. Usually, when inspecting for small flaws like fatigue cracks, it is preferable to use the wet method with fluorescent particles. The indications are then viewed under ultraviolet illumination in a darkened area. Otherwise, colored particles (red or black) are used.

2. The Magnetization Problem

In designing a magnetic particle test, the inspector must choose a combination of the various options described above. There are, fortunately, several excellent summaries(1-5) on methodology to aid him in reaching informed decisions on most questions. On the other hand there is one critical area in which the existing literature is of little help. This area concerns the specification of proper levels of magnetization and methods for determining that a specified magnetization is achieved. The reason that this question is critical is that if the magnetization is too low, the leakage field will not be strong enough to form a detectable indication. If, on the other hand, the field is too strong, indications will form at local saturations and minor perturbations due to surface strains, surface roughness, etc., thus giving false indications.

The only generally accepted rule for achieving proper magnetization is an empirical formula developed several years ago for the inspection of long narrow pieces, such as rods, by the coil magnetization method.(1-5) Even this rule, which gives the recommended number of ampere-turns as a function of the specimen dimensions, is, however, subject to question, because it does not allow for the fact that permeabilities, and thus flux densities, vary from one material to another in a given magnetizing field. When the extra factor of complex specimen geometry is added, the empirical rule is of little use except, perhaps, as a starting point for experimentation with a given specimen.

As a result of this situation, there are numerous empirically derived rules and procedures for determining how a specimen should be magnetized. Some of these results are published in the open literature, but many others exist only as guidelines used by various organizations concerned with magnetic particle testing.

The recent work of Kifer and Semenovskaya[6] serves as an example. From their experiments with artificial flaws in several steels with widely varying permeabilities, they derived empirical relationships for the recommended applied field strength as a function of flaw size and the coercive force of the material. However, they were unable to develop any well-defined relationship for natural flaws, even for similar materials.

In another study, Gregory et al.[7] varied several parameters, including specimen geometry, and concluded that industry-accepted inspection standards for required field strengths can be inadequate in certain regions of complex-shaped pieces. They also questioned magnetic particle equipment suppliers and users in the aircraft component industry about magnetization standards, and found considerable variance in the guidelines and procedures used at different installations.

Finally, in an evaluation program conducted by the Air Force Materials Laboratory[8], 24 parts with widely varying geometry, most containing detectable flaws, were sent to eleven different organizations for magnetic particle inspections by whatever method each organization considered best. The results varied widely, with only one group finding more than 90% of the flaws, while most of the other participants found fewer than half. It was concluded that, although most groups could substantially improve their performance by properly applying state-of-the-art knowledge of test principles, existing methods for determining magnetization requirements are inadequate.

From studies such as those just described. it is clear that the most serious obstacle to improved performance in magnetic particle inspection is the absence of well-founded specifications on magnetization requirements. It is also clear that the specifications needed cannot be as simple as the one coil magnetization formula that presently exists. Thus, one should expect that improved guidelines will take into account, to the extent necessary, flaw type and size, the magnetic characteristics of the material, and other test parameters such as the method of magnetization and method of particle application.

B. Analysis of the Magnetization Problem

The objective of the present program was to develop a research plan leading to the improved magnetization guidelines described above. Actually, there are two parts to the problem, as we noted earlier. The follow-on research program must first provide a basis for determining what the magnetization or near-surface flux density should be in a specific test, and, in addition, the research program should lead to a measurement technique that an inspector can use to check the level of magnetization as a pre-inspection step. Accordingly, the specific aims of the present effort were to plan research leading to (1) a predictive model that can be used to determine acceptable ranges of flux densities under various test conditions, and (2) instrumentation for measuring flux densities.

Regardless of how one chooses to approach the development of a predictive model, it is clear that there are three basic problems that must be addressed. The first of these is the definition of a detectable indication in terms of some measurable physical characteristic such as the total number of accumulated particles, the dimensions of the indication, or, perhaps the difference between the particle density at a flaw and the density in a flaw-free region. The reason that this is needed is because the prediction of detectability is the ultimate purpose of the model, and a mathematical model can do no more than produce data on the distribution of accumulated particles. One must, therefore, have some quantitative criteria by which to judge whether the predicted distribution data indicate detectability.

The second essential element of the model is the ability to predict accumulated particle distributions in a given inhomogeneous magnetic field. In this case there are several requirements that must be met. First, because most inspection procedures involve time dependent magnetization "shots", either AC or pulsed DC, it is essential that the predictive model provide an adequate treatment of the time dependent diffusion of particles in such fields. In addition, because experience indicates that the particle agglomeration process is quite different in the wet and dry methods of application, and appears to depend on the shape, magnetic characteristics and size distribution of the particles[4], these factors must also be treated in the predictive model. Finally, because analyses[9] of a similar process, namely, the formation of powder patterns in magnetic domain observations, indicate that interparticle interactions play an important role in the formation of an indication, predictions of the effects of such interactions should form a part of the particle density model.

It is, of course, impossible to treat all of these variables exactly in the development of a model. Fortunately this is not necessary because the only purpose of the model is to determine whether a detectable indication is formed in a given field. Thus, approximate models of these various effects are perfectly acceptable as long as the end result yields consistently correct predictions of detectability in fields typical of those encountered in magnetic particle flaw detection.

The third and final element of the predictive model concerns the determination of flux leakage fields associated with flaws. Here again there are a number of factors to be considered. There is, for example, ample empirical evidence that indicates that the intensity of the leakage field depends strongly on the magnetization characteristics of the specimen as well as on the magnitude of the unperturbed near-surface flux density in the specimen[6, 10]. It is also evident that the geometrical characteristics of the flaw (size, shape and location) play a role, and that the condition of the material in the vicinity of the flaw (e.g. existence of strains, surface roughness, etc) will have some influence on the leakage field. Finally, in addition to providing an adequate treatment of the leakage associated with a flaw, the predictive model must account for stray fields, such as those resulting from edges or other geometrical discontinuities in the specimen, because such fields also produce indications that interfere with flaw detection.

Turning now to the question of instrumentation for measuring the unperturbed flux density, there are, again, several requirements that must be satisfied. Because the sensor is to be used under shop conditions to measure flux densities in complex geometries, it must be easily portable, simple to operate, and capable of accurate positioning and measurement in "tight" configurations, such as near corners and in slots or notches in a specimen. Also, because the flux leakage intensity is greatest when the flux lines are perpendicular to the major dimension of a flaw, it is important that the sensor be capable of determining the direction, as well as the intensity, of the flux density on each segment of the surface to be inspected.

Our objective in the present project was, therefore, to plan a research program leading to the development of a predictive model and instrumentation that satisfy the requirements listed above. This was accomplished in three steps. First we used the facilities of the Nondestructive Testing Information Analysis Center (NTIAC) to survey the literature on magnetic particle testing, magnetic flux density predictions and measurements, and related topics. Next we conducted a critical review of methods identified through this search to determine which, if any, of the methods or devices presently available might be applicable to the magnetization problem. As an important part of this methods evaluation task, we also examined proposed methods, or methods not yet adequately tested, and assessed the likelihood that such methods would, with further development, prove useful in realizing the long-range objections of the follow-on program. Once this was done, we were able to formulate a systematic research plan that makes full use of available techniques, and is aimed at developing improved methods, as needed, for predicting magnetization requirements and measuring flux densities in pieces to be inspected. The results of this effort are reported in the next three Sections of this report.

C. Results of the Literature Survey

As was indicated above, the computerized bibliographic services of NTIAC were used as our primary source of reference material. In addition to the NTIAC file on nondestructive testing, we also searched the Defense Documentation Center TR (Technical Report) file, the National Technical Information Service file, and, through the commercial DIALOG service, compilations of material from Physics Abstracts and Engineering Index. From the NTIAC survey some 1600 abstracts were identified and reviewed, and about 120 articles were chosen for closer examination. Other sources included a survey of the magnetic particle testing literature published prior to 1965, several textbooks on magnetic field calculations conference proceedings on magnetic field calculations in electrical machinery and particle accelerator design, and references cited in the articles chosen for review.

The most useful articles located by this survey are referenced at appropriate points in Chapters II and III of this report. There are, however, a few review articles and one book on magnetic particle testing that are representative of the present state of development of the subject, and therefore deserve special note. These are the following:

R. C. McMaster, "Nondestructive Testing Handbook", Ronald Press, New York (1959).

H. J. Bezer, "Magnetic Methods of Non-Destructive Testing", British J. of N.D.T., pp. 85-93, Sept. 1964 and pp. 109-122, Dec. 1964.

American Society for Metals, "Metals Handbook, Vol. II", pp. 44-74, 1976.

C. E. Betz, "Principles of Magnetic Particle Testing", Magnaflux Corp., Chicago (1967).

Also of general interest is the following article reporting on a Soviet program to develop new empirical guidelines for magnetic particle inspection:

I. I. Kifer and I. B. Semenovskaya, "New Magnetic-Particle Methods of Inspection", Sov. J. NDT 8, 161-164 (1973.

Regarding the specific problems of defining detectability and developing a suitable particle diffusion model, we found little useful information in the literature. On the detectability problem, in particular, we found no data that would enable one to define a detectable indication in terms of some quantitative characteristic of the accumulated particle distribution. The situation regarding particle diffusion and agglomeration is somewhat better, in that we found a few articles on approximate methods of analysis,[9, 11, 12] and other material dealing with basic diffusion theory,[13] which should prove useful in developing an improved model.

As one might expect, there is an abundance of literature on methods that might be used to predict the magnetic field distributions associated with flaws in magnetized specimens. The techniques employed range from the very simple, approximate mathematical model proposed by Zatsepin and Shcherbinin,[14] and used extensively by workers in the Soviet Union, to elaborate computer programs developed mostly in this country and in England for magnet design applications.

Because of the great number of mathematical techniques, and combinations thereof, that one might use in developing a model suitable for the present purpose, we have not attempted to compile a complete, detailed list of all possible approaches. Instead, our review of the literature on this subject consists of a description of three very general classes of methods, along with examples from the literature, and comments on the applicability of each class of methods to the prediction of flaw leakage fields. In this review, which is presented in Appendix A, all potentially useful methods are treated as belonging to one of the following groups:

Classical Methods. This group comprises separation of variables techniques, image methods, and complex potential methods, i.e., all of the traditional approaches to obtaining exact mathematical solutions to problems in potential theory.

Approximate Methods. Under this heading we include mathematical models that involve simplifying assumptions or approximations regarding the mathematical properties of the magnetic field distribution. The surface charge model of Zatsepin and Shcherbinin is a leading example of such methods.

Numerical Methods. This class of methods includes computer programs for numerically solving the magnetic field problem by finite element or iterative techniques.

D. Methods Evaluation

In this Section we describe the results of a critical review of methods identified in the literature survey. Our purpose here was to determine to what extent available methods might be used and to define problems on which more research is needed.

1. Detectability

In our survey of the magnetic particle testing literature we were unable to find any generally accepted, quantitative criteria that define a detectable indication in terms of particle density or dimensions of the indication. Since the final mathematical model of the inspection process will predict particle density distributions for the purpose of determining detectability, it is essential that quantitative detectability criteria be defined.

One approach to the solution of this problem might be to simply accept certain visibility standards and perform magnetic particle experiments to relate these standards to the physical characteristics (particle density and dimensions) of a magnetic particle indication. Alternatively one might measure the brightness of a fluorescent particle (or some quantity related to optical contrast in the case of colored particles) and use this information to calculate the characteristics of an indication required for detectability. In either case, it will be necessary to check the results by performing inspection tests on real flaws.

2. Particle Diffusion

The only available mathematical model pertaining to magnetic particle density in an inhomogeneous magnetic field is that proposed by Kittel[11] in connection with the theory of powder patterns for magnetic domain studies. This model is based on the assumption that the particles are in thermal equilibrium with the medium in which they are suspended. There are actually two versions of the theory, depending on whether the particles are considered permanent dipoles of magnetic moment, m_o, or permeable, but unsaturated, bodies of susceptibility χ. The resulting expressions for the density at a point where the field strength is H are

$$\frac{n(H)}{n(0)} = \frac{\sinh(m_o H/kT)}{m_o H/kT}$$

and

$$\frac{n(H)}{n(0)} = e^{\chi H^2 v/2kT}$$

where v is the particle volume and n (0) is the density in the absence of a field.

From these results it is evident the density depends very strongly on the magnetic state of the particles because in one case the density varies exponentially with H while in the other case there is an exponential dependence on H^2. Since information on the susceptibility and size distribution of magnetic particles is considered proprietary by the suppliers of such particles, it will be necessary to measure these characteristics in the follow-on program to define the field strength conditions that determine which of the two equilibrium solutions is applicable.

While the use of an equilibrium theory has been justified for the particle sizes used in domain research(12), the fact that larger particles are used in magnetic inspection tends to cast some doubt on its validity as a predictive model in the present context. To investigate this question we used Stokes' formula for a sphere falling in a viscous fluid to calculate the kinetic energies of particles falling in air and in a liquid (the viscosity of water was used). These energies were compared with the energy of thermal motion (3/2 kT) at room temperature to determine whether the particles can reach thermal equilibrium in the gravitational field alone. The results are listed below -

Particle Diameter in μm	Energy of Free Fall/Thermal Energy	
	Air	Water
2	1.96	7.8×10^{-4}
4	251	0.10
10	1.5×10^{5}	61.7
20	2×10^{7}	7.9×10^{3}

The energy ratios clearly show that unless the particles are extremely small, perhaps less than 1 μm in diameter, thermal equilibrium is not realized in air. In fact, because the mean diameter of particles used in the dry inspection process is probably greater than a few microns, the effects of Brownian movement (thermal energy) is negligible. Thus while the equilibrium model is almost certainly invalid in this case, the fact that thermal motion is negligible suggests that a straightforward classical trajectory analysis (with viscous forces included) might be all that is needed to predict the initial distribution of particles on the surface of a magnetized specimen. To complete the analysis of the dry process, one might then compare the viscous force exerted by the stream of air used to remove excess particles, with the binding forces exerted by leakage fields to determine the distribution of particles that remain bound to the specimen.

Turning now to the wet process, where the particles are suspended in a medium with viscosity similar to that of water, the energy ratios listed above show that the validity of the equilibrium model depends strongly on the particle size distribution. While we were unable to obtain definitive data on size distributions from suppliers, one source did tell us that most of the particles used in the wet process have major dimensions in the 2 μm to 5 μm range. If this is the case, then the energy ratios indicate that the equilibrium model is probably adequate as a first approximation to the particle density in a static field. However, if we are to test the validity of this model and extend it to the treatment of pulsed or alternating fields, it will be necessary to develop a model that accounts for time-dependent departures from equilibrium. One way that this might be done is outlined in Appendix B.

3. Magnetic Field Calculations

In Appendix A we outline a few of the many methods that one might use to predict the leakage field associated with a near surface flaw. One of the simplest of these is the model proposed by Zatsepin and Shcherbinin[14] which, as we show in the Appendix, is based on the assumptions of uniform permeability and constant magnetization over the surface of the flaw. The resulting expression for the magnetic scalar potential is

$$\phi(\vec{x}) = \phi_0(\vec{x}) + \frac{M_s}{4\pi} \int \frac{\vec{e}_x \cdot \vec{n}\, ds}{|\vec{x} - \vec{x}'|}$$

where ϕ_0 is the potential associated with the applied field, $\vec{e}_x$ is the unit vector in the direction of magnetization (assumed constant), $\vec{n}$ is the unit normal to the surface at the point $\vec{x}'$ on the surface of the flaw, M_s is the magnitude of the magnetization at the surface, and the integral is over all points $\vec{x}'$ on the surface of the flaw. The corresponding expression for the field strength is easily obtained from the relation

$$\vec{H} = -\nabla\phi$$

It would indeed be fortunate if a model as simple as this should prove to be adequate for the analysis of accumulated magnetic particle densities. The model is, however, based on some rather drastic approximations and should therefore be subjected to extensive testing by comparisons with experimental data. Some work along these lines has been reported, and the results obtained to date indicate that the model works well in some cases but is deficient in others.

As an example, Novikova, et al,[15] performed a series of experiments with artificial flaws (slots) in iron and steel and, from these data, derived a semi-empirical expression for the unknown quantity M_s in terms of the permeability and applied field strength. They then demonstrated that with the value of M_s thus determined, excellent agreement was obtained between field strengths predicted by the equations given above and experimentally measured field strengths.

Some of their data are shown in Figure 2. Here H_x is the component of H in the direction of the applied field, which was parallel to the surface of the specimen, and H_y is the component perpendicular to the surface. While this agreement is encouraging, it must be remembered that the theoretical curves in this case involve a semi-empirical element, and this tends to leave open the question of how well the model will work when applied to other materials and other types of flaws.

There have, indeed, been other comparisons with experiment where the theory was found inadequate. In one case it was shown that serious disagreement was obtained at points just below the surface inside an open slot, and that it was necessary to add another term to the potential formula to account for the nonvanishing divergence of the magnetization (the second integral term in Equation (4) of Appendix A) in the region near the slot.[16] In still another series of experiments[6] it was noted that the strength of the leakage field is sensitive not only to the nominal permeability, $\mu = B/H$, but also to the differential permeability, $\mu_d = dB/dH$. Quite clearly, this observation cannot be explained with the Zatsepin-Shcherbinbin model which is based on the assumption of constant permeability.

Still another difficulty with the simple theory is illustrated in Figure 3. Here we show experimental data obtained at Southwest Research Institute on the vertical component of the leakage field associated with a test flaw in a hardened steel piston pin. The test flaw was produced in the surface of the piston pin by striking a hammer blow on a small diameter cone-shaped hardened steel pin which indented the surface. Such a flaw not only is a physical discontinuity or non-magnetic void, but also involves plastic flow as the indenter forces material to be displaced. As can be seen in the records of Figure 3, the signature (near the center of each record) is a complex function of the applied magnetic field: starting at the top record with the piston pin essentially demagnetized and with the piston pin essentially near magnetic saturation in the bottom record. (The signatures outside the middle two inches are caused by general work-hardening from service and also end effects.) Note in the first record that the signature (from left to right) first peaks upward, has a small perturbation near the middle, peaks downward, and returns to the baseline. This signature has a peak-to-

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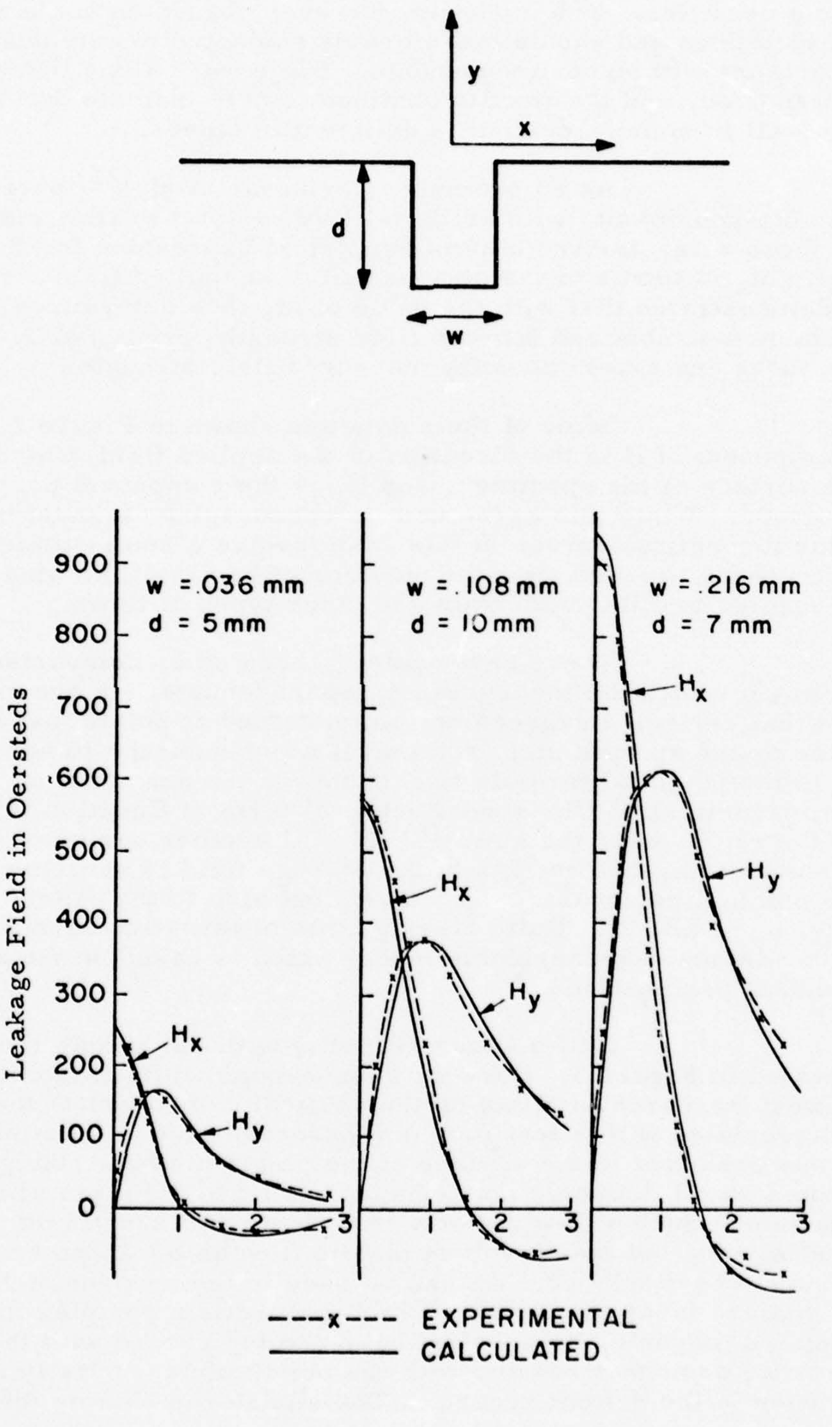
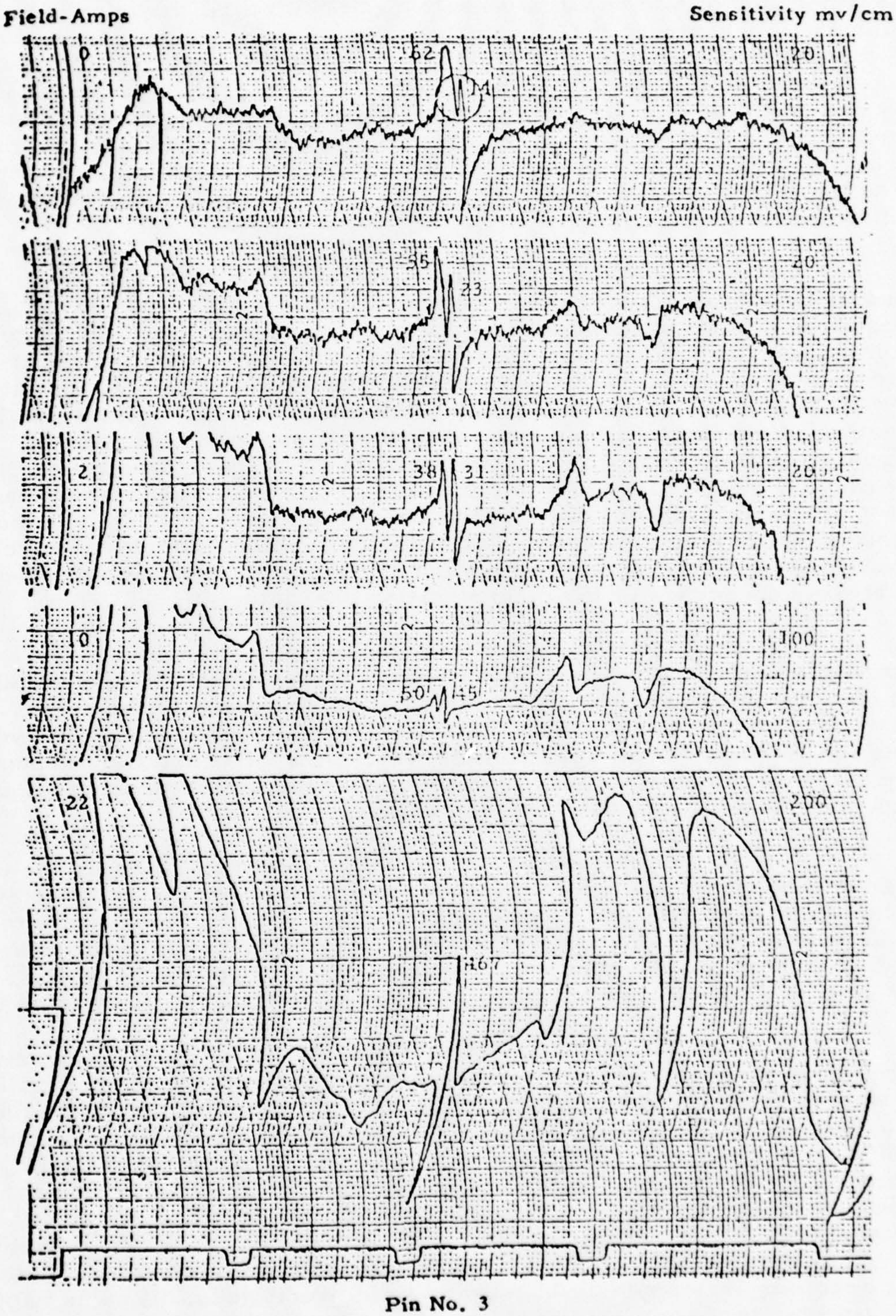


FIGURE 2. CALCULATED AND EXPERIMENTALLY MEASURED LEAKAGE FIELDS FOR SLOTS IN STEEL (Ref. 15)



Pin No. 3

FIGURE 3. CHANGES IN MAGNETIC PERTURBATION SIGNATURE AS INFLUENCED BY APPLIED MAGNETIC FIELD

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peak amplitude of 62 mv and a peak-to-peak horizontal distance of four divisions. By contrast the signature near saturation peaks downward and then upward (opposite to the signature of the top record); has a peak-to-peak amplitude of 467 mv and the peak-to-peak separation is only one unit, yet the flaw, the pin, the experimental equipment, the scan path, etc., are identical with record one; only the magnetization is changed. Obviously, the inspection results are not a simple function of the magnetization and therefore cannot be explained by the Zatsepin-Shcherbinin model.

On the basis of comparisons with experiments such as those just described, it would seem that the major deficiency of the model is its failure to account for variations in the permeability of the material in the immediate vicinity of a flaw. Since experiments of interest are almost always conducted at magnetization levels where the permeability is field-dependent, these changes in μ are due, in part, to the spatial variation of the field strength near a flaw. However, in the case of fatigue cracks or other flaws where there is significant plastic flow in a region around the flaw, the fact that the magnetization characteristic is strain dependent introduces an additional complication. Thus it is quite possible that corrections to the Zatsepin-Shcherbinin model will have to account not only for the field dependence of the permeability, but also for the strain dependence of the magnetization characteristic.

One way that this might be done is described in Appendix A under the heading "Corrections to the Surface Charge Model". The idea described here is an extension of the Zatsepin-Shcherbinin theory that retains much of the simplicity of the original model and yet takes into account, in an approximate way, the spatial and field dependence of the permeability. Although this approach seems promising, it is a new idea that will require further research.

Another approach would be to simply abandon the Zatsepin-Shcherbinin model altogether, and rely instead on an adaptation of one of the currently available magnetic field computer programs to the prediction of leakage fields. While this is certainly a valid approach, and one that is worthy of further consideration in the follow-on program, there are two very important reasons why we hesitate to recommend it.

First, as is evident from the description in Appendix A, computer programs that are capable of handling nonlinear magnetization in complex geometries are necessarily quite complicated, expensive to use, and, in the final analysis, provide little physical insight as to the significance of the various parameters involved in magnetic particle testing. The second reason is that at the present time, since we have no data on what constitutes a detectable indication, nor do we have a diffusion model that can tell us what effects errors in the leakage field will have on the particle distribution, there is no way that we can determine what accuracy is acceptable in leakage field predictions. It could happen, for example, that a simple extension of the Zatsepin-Shcherbinin theory, such as that proposed in Appendix A, is all that one needs in the way of a leakage model to consistently give correct predictions of detectability. On the other hand, further analysis might show that a more elaborate theory is needed, in which case one would either develop further corrections to the simple theory or, perhaps, abandon it in favor of a full-scale computer model. The main point is, however, that because we cannot at this time specify what accuracy is needed, there is no way to justify the use of a complex mathematical model in the initial stages of the follow-on program.

Given this situation, it seems to us that the best way to proceed is to first find answers to questions regarding accuracy requirements and then test flux leakage theories, beginning with the simplest model, to determine what corrections, if any, are needed. This, then, is the central theme around which our recommended research plan is structured. Details are given in Chapter III.

4. Magnetic Field Sensors

The final step in the evaluation phase of the present program was to determine a suitable approach for measurement of flux density in a material by an external sensor. Methods examined involved the use of Hall effect generators, magnetodiodes, vibrating or rotating coils, and eddy current techniques. Magnetodiodes, while useful in the measurement of weak fields, do not appear practical in the present case because of their strong nonlinear response at field strengths of interest.[17] Vibrating or rotating coils were also eliminated because they are usually difficult to use in complex geometries.[18] Eddy current techniques, on the other hand, appear promising since it is possible to obtain a flux determination inside the material without effects from stray magnetic fields in the air.[19] It appears, however, that additional development work is required on this technique primarily to determine the effects of complex geometry on the calibration. This leaves, as the most straightforward measurement approach, the use of Hall effect generators (Hall probes).[1]

To use a Hall probe for magnetic flux measurement in a material, the probe would be placed close to the specimen surface and oriented in a direction to measure flux tangential to the surface. The magnitude of the tangential value would be multiplied by the specimen's permeability ($B = \mu H$) to determine the flux density in the material.

One requirement that a Hall probe must satisfy in this application is that the element be as close as possible to the surface since, in the case of a sharp field gradient (such as is encountered at a corner), the field measured above the surface will be different from that at the surface and thus the accuracy of the internal field calculation will be affected with liftoff. Just how close to the surface the probe must be will have to be determined by experiment.

Several commercially made probes are available which have a minimum liftoff. One probe[20] has an element size of 0.120 x 0.060 in. with a 0.004 in liftoff. Sensitivity is 0.06 $\frac{V}{A \cdot KG}$. (Awaiting quote on price).* Another probe[21] has a 0.030 in. diameter active element with a liftoff of 0.010 in. and sensitivity 0.11 $\frac{V}{A \cdot KG}$. Price is $150. A third probe[22] contains an element measuring 0.078 x 0.187 with a liftoff of 0.015 in. Price is $390. Also available are nonencapsulated active elements measuring 0.040 x 0.090 in. While these are too fragile to be used in this form, it may be possible to encapsulate them with one end placed very close to the edge. Due to the relatively large size of all the above elements, measurement error may be introduced since even if the element is adjacent to the specimen surface, the top portion of the element will be measuring flux which is far removed from the surface.

SwRI has the capability of manufacturing Hall probes with sensor elements measuring only 0.001 x 0.004 in. or 0.005 x 0.015 in. both with a 0.010 liftoff. (Sensitivity is 0.05 $\frac{V}{A \cdot KG}$.) While this liftoff is somewhat greater than that of commercially available probes, the much smaller element size may reduce errors obtained with larger elements. Production of several SwRI probes would require 6-8 man weeks.

An evaluation was also made of techniques for making high resolution measurements of the magnetic leakage fields around cracks, as such measurements will be required in experimental phases of the follow-on program. A survey of the literature showed that only specially constructed Hall probes[23] and vibrating wire loops were capable of sufficient resolution. The vibrating wire loop appears to be less desirable due to the complexity of the vibrating mechanism.

Measurement of crack leakage fields would require a differential probe arrangement to eliminate effects from the large applied field outside the specimen. For measurement of the normal magnetic field component, the probe could be placed as close as 0.0005 in. to the specimen surface and the two differentially arranged elements might be separated by 0.010 in. (This is the arrangement used in several SwRI studies of defect leakage fields.) For measurement of the tangential component a special probe should be constructed with the element as close as possible (0.010 in) to one edge. Otherwise, probe dimensions would be identical to the normal component probe.

* Footnote: V = volts, A = Amperes, KG = Kilogauss.

III. RESEARCH PLAN

The final task in the present project was the formulation of a research plan leading to the development of a mathematical model for predicting flux density requirements and instrumentation for measuring flux densities. In our analysis of research required for the development of a mathematical model we concluded that there are three areas where more work is needed. These are (1) the definition of detectability criteria, (2) the development of methods for predicting particle distributions in known fields, and (3) the development of a method for predicting flux leakage fields associated with flaws. As far as flux density instrumentation is concerned, it was suggested that the use of a Hall probe to measure the tangential field strength at the surface of a speciment offers the most convenient solution. It was also noted, however, that more experimental work is needed to determine if commercially available probes are adequate. Our purpose, therefore, in this final Chapter, is to define a sequence of tasks that will resolve existing uncertainties and will result in the desired mathematical model and flux density instrumentation.

A. Possible Approaches to the Development of a Mathematical Model

Before presenting our recommended plan it is appropriate to review the various approaches that one might take in developing a model of the inspection process. In this section we will, therefore, discuss three general approaches, namely, the purely empirical method, where one relies entirely on experimental data and statistical analyses, the computer simulation method, where the entire process is modeled on a digital computer, and what we will call the analytical approach, which involves both theory and experiment.

In our comments on the advantages and disadvantages of these approaches we have in mind, in particlar, the large number of parameters that must be investigated in developing the model. As was noted in the Technical Discussion these include the following:

1. method of magnetization
2. method of particle application
3. particle size distribution and magnetic characteristics
4. magnetic properties of the specimen
5. flaw type and geometry
6. stray fields due to irregular geometry of the specimen, surface strains, etc.

1. Possible Approaches

Empirical. With this approach one would conduct magnetic particle experiments on a large number of specimens and attempt to fit the results with a semi-empirical (phenomenological) model. The specimen population and method of experimentation must include variations of all factors listed above. This is the approach being pursued, on a limited basis, by Kifer et al[6].

Computer Simulation. This approach would require the adaptation of an available finite difference program (or programs) to the treatment of flaw leakage fields and the preparation of a stochastic model of particle diffusion. It would then be necessary to check the adequacy of the overall program by comparison with experiment. Finally, one would perform computations for a large number of specimens and analyze the results as in the empirical approach.

Analytical. Here one would employ approximate methods of calculation to determine the significance of each factor separately as it effects detectability. The results would be checked by experimentation with idealized specimens, i.e., with specimens designed to check the assumptions and approximations used in the calculations. Next, models that include combinations of factors would be constructed and the results of calculations compared with experiment. The model would be simplified by disregarding factors that can be shown to play an insignificant role in determining detectability. Thus, the final model would treat only those variables that are important and would employ the simplest calculational technique consistent with the required accuracy.

2. Advantages of Each Approach

Empirical. This is the most direct approach to generating a data base for real materials and flaws.

Computer Simulation. The computer method is ultimately capable of the accurate solution of almost all problems in nonlinear magnetostatics and particle diffusion (limited by computer memory and cost constraints), and offers easier control over specimen parameters than the empirical approach.

Analytical. This approach is likely to substantially reduce the number of situations that must be analyzed by keeping as variables only those parameters that are important. It is, therefore, the most economical approach and offers analytical insight that may prove useful in the interpretation of magnetic particle and other flux leakage test results.

3. Disadvantages of Each Approach

Empirical. This approach is expensive because a large number (probably hundreds) of specimens must be prepared and characterized to provide a statistically reliable data base. Also, the control of specimen characteristics and determination of those characteristics is difficult.

Computer Simulation. Available programs must be modified to accommodate the three dimensional, nonlinear problems of interest here. However, the present state of development of such programs (see Appendix A) suggests that such modifications will pose serious computational difficulties. Thus, this approach may not be feasible at the present time. In any case, it will certainly be expensive, though probably not so expensive as the purely empirical approach. As far as staffing is concerned, it requires personnel with extensive experience in large-scale computer applications.

Analytical. This approach will probably require more calendar time (but at a substantially lower level of effort) than other approaches. It is also difficult to plan in detail because the decision as to how best to proceed at any given stage of development must be based on the results of preceding calculations and experiments. It requires an investigative team with high levels of expertise in analytical, computational and experimental solid state physics; close coordination between theorists and experimentalists is essential.

As we have already indicated, the large number of parameters, and existing uncertainties regarding the significance of these parameters, leads us to recommend the analytical approach, rather than the empirical or computer simulation methods. Our specific recommendations, including a task-by-task description of the research plan, are presented below.

B. Recommended Research Plan

The recommended research program consists of two major phases, one aimed at the development of a mathematical model of the magnetic particle inspection process, and the other leading to the development of instrumentation for measuring flux densities in specimens to be tested. In structuring the plan, our intent has been to concentrate first on those problems that relate to accuracy requirements, because the answers to such problems could provide a basis for considerable simplification of the mathematical model. Thus, at least in the first phase, it is important that individual research tasks be performed in the order indicated.

Phase I

The list of tasks that comprise Phase I is given in Figure 4. A description of each task follows.

1. Determine Particle Density Requirements

The objective of this task is to define a detectable indication in terms of some quantitative property such as the dimensions of the indication or the number of accumulated particles.

In the case of fluorescent particles, it is likely that this can be accomplished by means of Michelson's definition of visibility, V, which is[24]

$$V = \frac{Imax - Imin}{Imax + Imin}$$

PHASE I

RECOMMENDED RESEARCH PLAN

1. Determine particle density requirements.

2. Develop particle diffusion model.

3. Apply diffusion model to the determination of leakage field accuracy criteria.

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4. Test simpler leakage models by comparison with experiment.

5. Determine significance of flaw geometry, permeability and other test parameters.

6. Develop corrections to leakage model.

7. Combine leakage and diffusion models.

8. Test combined model by comparison with magnetic particle experiments.

9. Conduct parameteric study to determine inspection guidelines.

FIGURE 4. RECOMMENDED RESEARCH PLAN FOR THE DEVELOPMENT OF A MATHEMATICAL MODEL

where Imax is the maximum intensity in a luminous field and Imin is the minimum intensity. Once the brightness of a single fluorescent particle is known one can relate the minimum and maximum intensities to particle densities, thus completing the definition of V in terms of particle density variations. With nonfluorescent colored particles the situation is more complicated, because it involves considerations of color contrast, but probably can still be handled in much the same way. In either case, and regardless of how one chooses to define detectability, it will be necessary to experimentally verify whatever criteria are chosen by means of magnetic particle tests.

2. Develop Particle Diffusion Model

This is a major task that involves both theoretical and experimental elements. The first step (a) is a theoretical development while (b) involves comparisons with experiment. If the diffusion model, as originally formulated, fails to account for experimental observations, it will then be necessary to refine the model and repeat the comparisons until satisfactory agreement is realized. Some specific suggestions concerning experimental methods are given in Appendix C.

(a) The objective of the theoretical effort is to develop a method for calculating accumulated magnetic particle densities in a given field $H(\vec{x}, t)$. The field should be typical of those produced by surface flaws and should include, in addition, a stray field component representative of leakage from minor surface perturbations such as surface roughness. It will also be necessary to consider the effects of strong but slowly-varying fields due to leakage from edges of the test piece. Time dependence should include AC, DC and half-wave rectified fields.

(b) Next, one should verify the adequacy of the model by comparison with magnetic particle experiments. It will be necessary to experimentally measure the components of $\vec{H}$ in the vicinity of a surface flaw and at points above a flaw-free surface. Experiments should also include field measurements and observations of accumulated particle densities with smooth and rough surfaces and with flaws near edges. The diffusion model will be considered adequate if it can successfully account for the detectability of flaws by the magnetic particle method.

3. Determine Accuracy Criteria

The intent here is to determine the range of values of the parameters that characterize the near surface field within which magnetic particle flaw detection is assured. In so doing, one will also determine the accuracy requirements for leakage field predictions. Again, both theoretical (a) and experimental (b) work is required.

(a) The first step would be to apply the diffusion model developed in Task 2 to the calculation of acceptable ranges of values of flaw leakage fields, leakage field gradients, and stray fields as a function of the method of magnetization (AC, DC or half-wave rectified). Acceptable values in this case are those that result in detectable indications as determined by the criteria defined in Task I.

(b) Next, one would conduct magnetic particle experiments as needed to verify the conclusions reached in (a).

4. <u>Test Flux Leakage Models</u>

(a) The first step in this task would be to perform leakage field calculations for simplified models of cracks and subsurface inclusions using the surface charge model defined in Appendix A, as a function of the near-surface flux density in a flaw-free region.

(b) Next, one would experimentally measure the leakage fields associated with artificial defects (slots), actual fatigue cracks and subsurface inclusions as a function of the strength of the unperturbed flux density.

(c) The final step would be a comparison of the results obtained from Tasks 4(a) and 4(b) to determine the extent to which the simple methods of calculation are useful. Where discrepancies exist, additional calculations and/or experiments would be performed to determine the reason for the disagreement.

5. <u>Determine Significance of Test Parameters</u>

On the basis of the calculated and experimental data from Task 4, it should be possible to determine the relative importance of flaw geometry, variable permeability, surface roughness, etc. in the determination of near surface fields. Also, one should attempt to estimate, on the basis of these comparisons between theory and experiment, the errors introduced by the simplifying assumptions inherent in the surface charge model. In general, the intent here is to determine where improvements in the calculational model are needed.

6. <u>Develop Corrections to the Leakage Model</u>

(a) Based on the conclusions drawn in Task 5, one should next develop corrections to the surface charge model.

(b) After this is done, leakage fields should be recalculated using the improved model and compared with the experimental results obtained in Task 4(b). The results of these comparisons would then be analyzed as in Task 5. If further improvements are needed, the required corrections should be developed and the calculations and comparisons with experiment repeated.

7. Combine Leakage and Diffusion Models

Next one would combine the particle diffusion model developed in Task 2 with the leakage model developed in Tasks 4 through 6. The result will be a mathematical procedure for which the input data will be the near-surface flux density in a flaw-free region, the method of magnetization (AC, DC, etc), the method of particle application (dry or wet), and whatever flaw geometry and materials data are required (this will be determined in Tasks 4 through 6). The output will be predictions of the near surface field parameters of significance in a magnetic particle inspection, the accumulated particle density, and the detectability parameters defined in Task 1. If the values of these detectability parameters fall within the acceptable range, as defined in Task 1, then the model will indicate that the flaw is detectable, and that the value chosen for the flux density at the surface therefore corresponds to an acceptable level of magnetization.

8. Test Combined Model

The final step in the development of the mathematical model is to verify its accuracy in the prediction of detectability by comparison with the results of the magnetic particle experiments performed in Task 2(b). Supplementary experiments may be performed as needed to fully check the reliability of the model.

9. Parametric Study

Once it has been determined that the combined mathematical model yields reliable predictions of detectability it should be applied to the prediction of flaw detectability as a function of the unperturbed flux density at the surface, flaw type and geometry, method of magnetization, and the magnetization characteristics of the material, for several typical flaws in materials of interest. Finally, the data thus generated may be assembled in tabular form to show acceptable ranges of flux desnities for the flaws and materials considered.

This concludes our plan for Phase I - the development, testing and application of a mathematical model of the inspection process. We now turn to the recommended research plan for Phase II, which is aimed at the development of flux density instrumentation.

Phase II

As was indicated earlier, our evaluation of methods for measuring the near-surface flux density in a specimen indicated that the most easily implemented approach would be that involving the use of a Hall probe to measure the tangential field just outside the surface to be inspected. If the value of the interior flux density is desired, then, because the tangential field is continuous across the surface, one need only multiply by the permeability to determine the tangential flux density inside the specimen.

The main point to be investigated in Phase II is, therefore, the accuracy of the tangential Hall probe method in determining interior flux densities. One would hope that commercially available probes will prove adequate, so that no further developmental effort is needed. However, because available probes have rather large sensing elements[20, 22], and, in the intended application, these probes might be used in situations where the field varies significantly over the area of the element (e.g., in measurements near a corner of a specimen), experimental tests of the method are needed. Our recommended plan for these tests is described below.

1. Sensor Selection

To minimize errors introduced by the displacement of the sensing element above the surface, and by the averaging effect of a large sensor area, probes with minimum liftoff and minimum sensor area should be selected from among those commercially available. To measure the internal flux density for comparison with the tangential probe data it will be necessary to employ a second probe, positioned in a small hole or slot located in the specimen near the surface. The sensor selected for this purpose may be either another Hall probe or a small coil. In either case, the overall size of this flux density sensor should be as small as possible, so that the dimensions of the hole, which will perturb the flux at the sensor, can be minimized. As some perturbation of the interior field by the interior sensor is inevitable, it will be necessary to apply calculated corrections to the flux density values thus obtained.

2. Specimen Design

Because the accuracy of the tangential probe method is likely to be influenced by the method of magnetization, specimen geometry and permeability, such parameters should be varied in testing sensors. It is particularly important that specimens be designed to accommodate tests in both uniform fields and in regions where the field is complex, such as near a corner. Irregular cylindrical specimens such as those illustrated in Figures 5 and 6 should suffice.

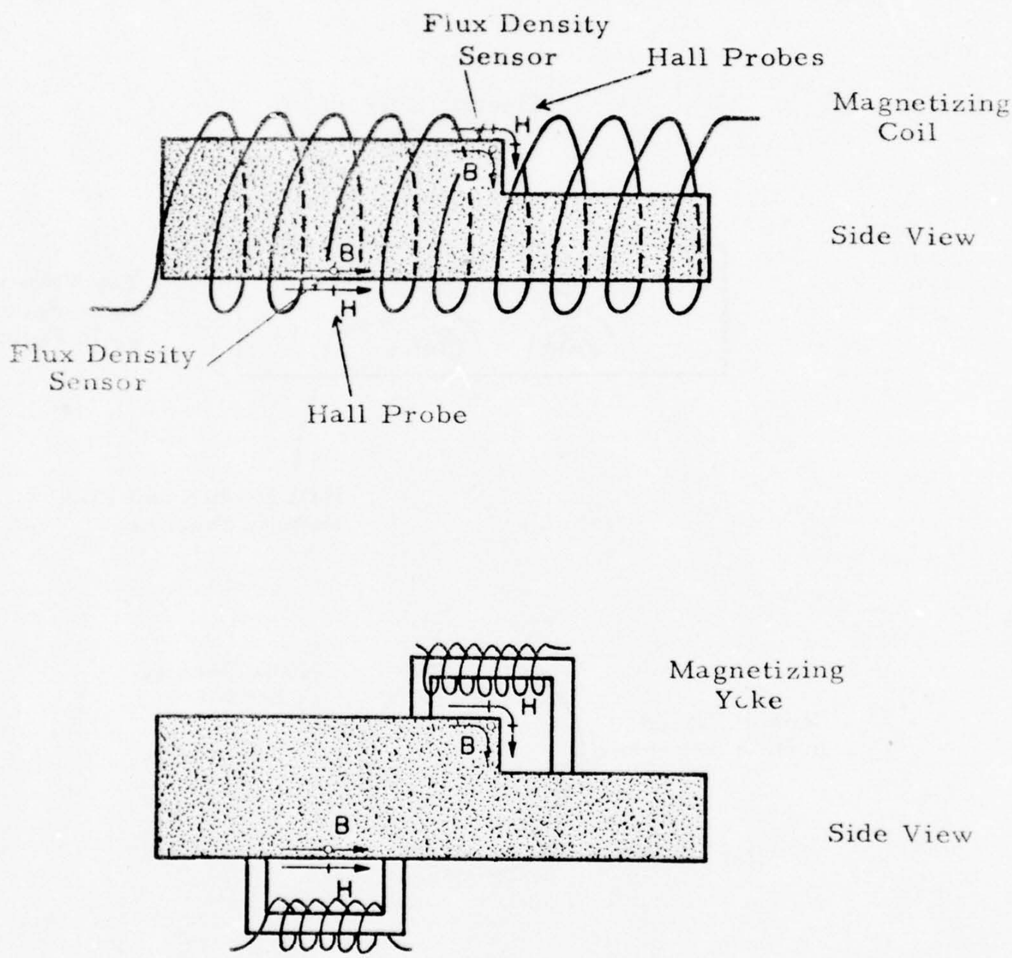

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FIGURE 5. EXPERIMENTS WITH TANGENTIAL HALL PROBES - COIL AND YOKE MAGNETIZATION

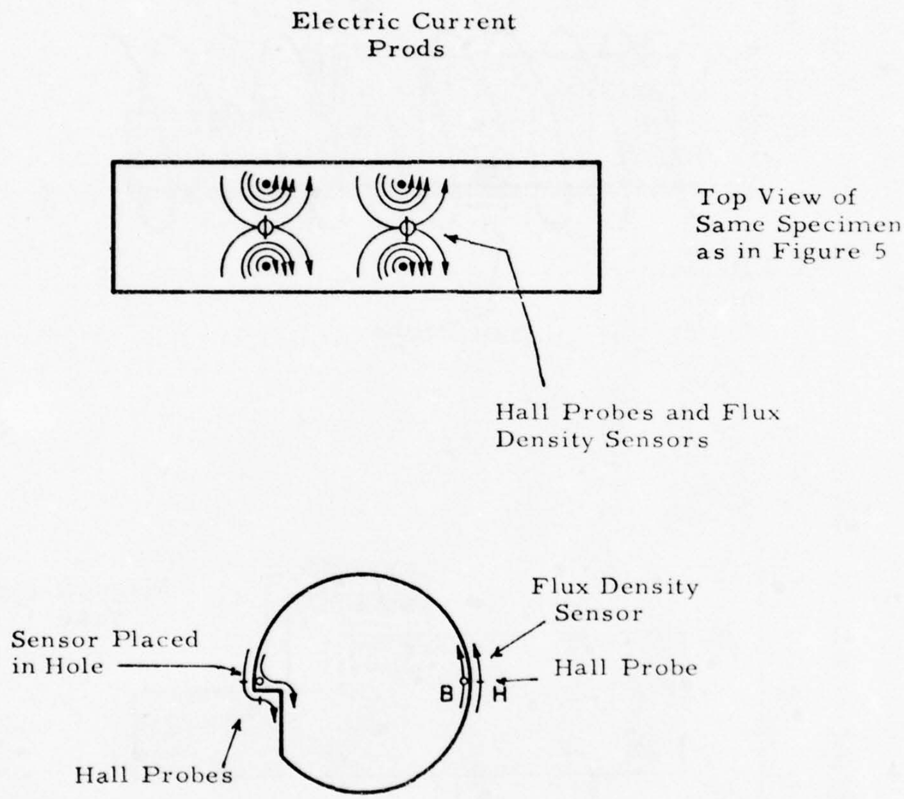

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FIGURE 6. EXPERIMENTS WITH TANGENTIAL HALL PROBES - CURRENT MAGNETIZATION

3. Experiments

Experiments with tangential probes and interior flux density sensors, such as are illustrated in Figures 5 and 6, should be performed to test the tangential probe method. In all cases, the magnitude of magnetizing field or current should be varied to test for possible effects of field dependent permeability.

C. Implementation of the Research Plan

As is evident from the preceding discussion, the recommended plan is a major effort involving several theoretical and experimental research tasks. Also, because a determination of the magnitude of some tasks, such as the development of corrections to the leakage model, must await the results of preceding tasks, the level of effort required is difficult to define. It is clear, however, that even if all uncertainties should be easily resolved, the entire program will be costly.

Because we realize that resources are limited, and that, for this reason, it is always difficult to support research programs at the desired level of effort, we have given some thought to alternate ways in which the recommended program might be implemented. These are discussed below.

1. Recommended Program

From the standpoint of efficiency, it is usually best to provide continuity in any research program. For this reason we suggest that the best way to implement the recommended research is to provide support for a minimum of two years at a minimum of 4 man-years total effort. With this level of effort we think it is realistic to set as a goal the completion of all tasks in this plan. However, it should be realized that the difficulty of some tasks is uncertain at the present time and that more effort may therefore be required.

2. Incremental Support

As an alternative to the recommended program one might consider a one-year program at a two man-year level. In this case the objective would be the completion of the first five tasks in Phase I and all of Phase II. The reason for this particular division of effort is that after completing the first five tasks in Phase I, accuracy requirements for the predictive model will have been defined, and the need for specific improvements in existing models will have been identified. Thus, at this point it should be possible to make realistic estimates of the effort required to complete the program.

3. Minimum Effort

Because we see no way to reduce the total effort required to achieve the long-range goals of the program, a minimum effort in this case would be a program in which the level of effort is reduced and the time frame extended. However, because several years may be required to produce the desired results, one may wish to apply existing models to the generation of interim guidelines for flux density requirements, with the understanding that such results are tentative. This would, in our opinion, be a useful additional step, because existing methods are certainly capable of producing better-defined requirements than are presently available.

In any case, regardless of how the program is implemented, there are two factors that should, in our opinion, be considered essential. These are the following: (1) the level-of-effort should be large enough to permit consideration of all test parameters that might influence detectability, and should also provide for comparisons of theory and experiment. (2) The scheduling of tasks should be such as to allow time for the analysis of accuracy requirements and the determination of the significance of test parameters, before proceeding with further development of the model.

APPENDIX A

METHODS FOR THE CALCULATION OF MAGNETIC FIELDS

In this Appendix we review mathematical methods that one might use to determine the magnetic field distribution in the vicinity of a flaw or other irregularity in a magnetized specimen. Because of the very large number of techniques that are applicable, to some degree, to the problem at hand, we will not attempt to describe each method in detail. Instead we have chosen to discuss three very broad classes of mathematical models and cite examples of their application from the literature.

The first class of techniques to be discussed comprises the so-called "textbook" methods of solution. These classical approaches generally lead to exact solutions to rather idealized problems and are, therefore, of limited usefulness in themselves for predicting complex fields like those encountered in magnetic inspection problems.

Next we will describe approximate methods, at least one of which has already been applied with some success to the analysis of leakage fields around flaws.

The third and final class of methods consists of the various numerical techniques that have been applied to magnetic field problems. Although most of these very elaborate computer programs were developed to aid in the design of magnets in particle accelerators and electrical machinery, many are equally applicable to the prediction of flaw leakage fields.

A. Classical Methods

1. Direct Solution of Poisson's Equation

The methods that fall in this category are those that lead to exact solutions to the scalar or vector potential equation by separation of variables or some equivalent technique. Such solutions are limited to a few simple geometries and can be applied only in linear (constant permeability) problems. However, recent work (discussed below) indicates that it may be possible to apply these methods to transformed potential equations in which the effects of variable permeability are accounted for in the definition of the potential.

An introduction to such techniques can be found in almost any advanced text on electromagnetic theory. Examples are

J. D. Jackson, "Classical Electrodynamics", John Wiley and Sons, New York (1962)

W. K. H. Panofsky and M. Phillips, "Classical Electricity and Magnetism", Addison-Wesley, Reading, Mass. (1955)

Numerous examples, both in the text and in the exercises, are given in

> W. R. Smythe, "Static and Dynamic Electricity", Third Edition, McGraw-Hill, New York (1968)

The recent development referenced above is described in a paper by S. G. Zaky and S. D. T. Robertson in the following report:

> "Proceedings of the Third Reno Conference on Magnetic Fields", Engineering Report No. 46, published by the Electrical Engineering Department, University of Nevada, Reno, Nevada (Sept. 1971)

These authors show that in the case of variable permeability, the substitution

$$\xi(\vec{x}) = \frac{\mu(\vec{x})}{\mu_0} \phi(\vec{x}) \tag{1}$$

where ϕ is the magnetic scalar potential, leads to Poisson's equation for the transformed potential ξ. Thus

$$\nabla^2 \xi = \frac{1}{\mu_0} \nabla \cdot \vec{M}' \tag{2}$$

where

$$\vec{M}'(\vec{x}) = \frac{1}{\mu(\vec{x})} \xi(\vec{x}) \nabla \mu(\vec{x}) \tag{3}$$

They then transform this equation into an integral equation for ξ which, they suggest, can be solved in the same way as the integral equation for in the linear problem. It might also be possible to apply other classical methods of solution, but this has yet to be investigated.

2. Image Methods

The image method is a very direct way to generate exact solutions in certain simple geometries, again with the assumption of constant permeability. The class of problems that can be handled is broadened considerably when the image technique is combined with complex potential methods described below.

The image technique is described in all three of the texts cited above. More detailed discussions, with numerous examples may be found in

B. Hague, "The Principles of Electromagnetism Applied to Electrical Machines", Dover, New York (1962) Chapter IV.

L. W. Bewley, "Two-Dimensional Fields in Electrical Engineering", Dover, New York (1963). Chapter 5.

C. Complex Potential Methods

The classical approach is limited to two dimensional problems and requires that the potential be constant on a given surface. This means that an exact solution for the flux in air is possible only if the magnetized body has infinite permeability. However, as the relative permeability in ferromagnetic materials is very large, the results obtained by assuming infinite permeability are often adequate, at least as a first approximation. Since the method can be applied to a variety of rather complex two dimensional geometries, it seems likely that the classical complex potential approach will prove useful, at least in checking the accuracy of approximate methods.

An introduction to complex potential methods is given in Chapter 4 of the text by Panofsky and Phillips, which was cited above. For a more extensive discussion see the text by Bewley (also cited above) or, for a more advanced treatment, Part III of the following book:

K. J. Binns and P. V. Lawrenson, "Analysis and Computation of Electric and Magnetic Field Problems", Pergamon Press, New York (1973)

Recent extensions of the classical approach to the analysis of two dimensional fields have shown that it is possible to include the effects of finite and variable permeability, although the analytic solution of the resulting equations has been attempted in only a few simple cases. (Numerical solutions will be discussed later.) The theoretical development is presented in the following articles:

K. Halbach, Nuc. Instr. and Meth. 64, 278 (1968) and 66, 154 (1968).

B. Approximate Methods for Complex Geometries

In this Section we depart from the usual format and instead examine one approximate method, the surface charge or Zatsepin-Shcherbinin model(14), in some detail. The reason for this is that the surface charge model is the simplest approximate theory applicable to complex geometries, and yet can form a basis on which successively better approximations are built. In our opinion, the use of this approximation, with corrections added as necessary, is the most economical approach to solving the flaw leakage problem.

The description that follows will be limited to the case where the specimen is magnetized by an external field (i.e. no currents flow in the specimen) because this is the only case to which this particular approximation is applied in the literature. There is, however, no reason why a similar approximate model cannot be formulated in terms of the vector potential, which is necessary if currents flow in the specimen.

1. Surface Charge Model

As is well known, one can express the magnetic scalar or vector potential in terms of surface and volume integrals involving the magnetization field. Thus, if one can by some means estimate the magnetization, an approximate solution for a body of arbitrary shape can be generated by direct integration. If, for example, one assumes uniform magnetization of a certain value, then the expression for the scalar potential reduces to a surface integral of the same form as the solution for the electrostatic potential in the presence of a charged body. This is the approach used by Zatsepin and Shcherbinin in their investigations of the influence of crack geometry on the leakage field. The theoretical development, in a form slightly different from that used by Zatsepin and Shcherbinin, is given below.

The starting point is the well-known expression for the magnetic scalar potential $\phi(\vec{x})$:[25]

$$\phi(\vec{x}) = \phi_0(\vec{x}) + \frac{1}{4\pi}\int \frac{\vec{M}(\vec{x}')\cdot\vec{n}\,dS}{|\vec{x}-\vec{x}'|} - \frac{1}{4\pi}\int \frac{\nabla_{\vec{x}'}\cdot\vec{M}(\vec{x}')}{|\vec{x}-\vec{x}'|}\,dV \tag{4}$$

where ϕ_0 is the potential associated with the applied field, $\vec{M}(\vec{x})$ is the magnetization and $\vec{n}$ is the normal to the surface of the magnetized body at the point $\vec{x}'$. The first integral is over the surface of the specimen and the second is over its volume.

The field strength $\vec{H}(\vec{x})$ is minus the gradient of the potential, i.e.

$$\vec{H}(\vec{x}) = -\nabla\phi(\vec{x}) \tag{5}$$

and the magnetization can be written as

$$\vec{M}(\vec{x}) = \frac{\mu(H)-\mu_0}{\mu_0}\vec{H}(\vec{x}) \tag{6}$$

where $\mu(H)$ is the field dependent permeability. Thus it can be seen that (4) is, in general, a nonlinear integral equation for the unknown potential $\Phi(x)$. In the general case, where the permeability varies with H, one must solve (4) numerically, i.e., there is no known analytic solution even in simple geometries.

However, if it is assumed that the permeability can be approximated by a constant, then, from (6),

$$\nabla \cdot \vec{M}(\vec{x}) = \frac{\mu - \mu_0}{\mu_0} \nabla \cdot \vec{H}(\vec{x}) = \frac{\mu - \mu_0}{\mu_0 \mu} \nabla \cdot \vec{B}(\vec{x}) = 0$$

and the second integral in (4) vanishes. If it is further assumed that the magnetization is constant, in both magnitude and direction, on the surface of the specimen, then (4) reduces to

$$\Phi(\vec{x}) = \Phi_0(\vec{x}) + \frac{M_s}{4\pi} \int \frac{\vec{e}_0 \cdot \vec{M}\, dS}{|\vec{x} - \vec{x}'|} \tag{7}$$

where M_s is the magnitude of the magnetization and $\vec{e}_0$ is a vunit vector in the direction of magnetization.

Equation (7) is what we call the surface charge model and is equivalent, in all respects, to the model proposed by Zatsepin and Shcherbinin. It is a very simple formula to apply because the integral involves only geometrical quantities, i.e., it depends only on the shape of the flaw. The only factor, aside from the known potential Φ_0, that depends on the strength of the applied field is the surface magnetization M_s. In most applications, where the primary interest has been in the shape of the leakage field, and not its magnitude, the value of M_s is not needed. On the other hand, in magnetic particle applications, magnitude is important and some attention must therefore be given to the determination of M_s.

One might assume, for example, that the magnetization is not appreciably altered by the presence of the flaw, in which case M_s is approximately equal to the magnetization just inside the surface in a flaw-free region. Then, given the value of the tangential component of the field strength (H_t) just outside the surface, an estimate of M_s would be

$$M_s \sim \frac{\mu(H_t) - \mu_0}{\mu_0} H_t \tag{8}$$

However, as this particular estimate of M_s has not been tested by comparison with experiment or with more accurate calculations, we have no basis for judging its usefulness.

In Section II. D of the text of this report, we describe comparisons with experiment that indicate that, despite the rather drastic assumptions of constant permeability and uniform surface magnetization, the surface charge model works surprisingly well in describing the major features of the leakage field around simulated cracks.[15] Detailed comparisons with experiment show, however, that the model is in need of refinement as discrepancies have been noted at points very close to the surfaces of simulated cracks.[16] Also, because the model is based on the assumption of uniform magnetization, it is inherently incapable of explaining certain other experimental facts, such as changes in the shape of the leakage field profile as a function of magnetizing current.

2. Corrections to the Surface Charge Model

To improve the surface charge model one must avoid the assumptions of constant permeability and uniform magnetization. This can be done by first recognizing that the "solution" for the magnetic potential in terms of the magnetization is not really a solution, but an integral equation, because the magnetization at a given point in the material depends on the field strength at that point and the field strength depends on the potential. Thus, to solve the potential problem with variable permeability one must solve the corresponding integral equation for the potential.

One very promising approach to the solution of the integral equation is based on the method of successive approximations. With this approach, one first calculates, by some approximate method, the potential inside the magnetized body. This approximation is then substituted for the unknown potential in the integral, and the resulting expression is evaluated to generate a second, improved approximation to the potential. The process is then repeated until the potentials generated in successive iterations agree to some predetermined accuracy. Experience with such iterative solutions indicates that the number of iterations needed is often very sensitive to the starting approximation. Thus, if the initial approximation happens to be a very good approximation to the exact potential, it may happen that only one or two iterations are needed. Therefore, while the approximate methods described above may not be adequate in themselves, they may prove to be invaluable in generating initial approximations for use in an iterative calculation.

As an example of how this approach might be applied to derive corrections to the Zatsepin-Shcherbinin model, suppose we let $\phi'(x)$ be the approximate solution given by (7). Then, from (5) we can calculate the approximate field strength,

$$\vec{H}'(x) = -\nabla \phi'(\vec{x}) \tag{9}$$

and from (6) the approximate magnetization

$$\vec{M}'(\vec{x}) = \frac{\mu(H'(\vec{x})) - \mu_0}{\mu_0} \vec{H}'(\vec{x}) \tag{10}$$

If we substitute this function on the right side of (4) and carry out the integration, the result will be an improved estimate of the potential which we might call $\phi^2(\vec{x})$. The process can then be repeated, if necessary, to generate still better estimates of $\phi(\vec{x})$.

To evaluate the volume integral in (4) one must compute the divergence of $\vec{M}$, which, according to (6), now involves the function $\mu(H(\vec{x}))$ as well as $H(\vec{x})$ itself. In principle, this causes no difficulty, as an elementary calculation gives

$$\nabla \cdot \vec{M} = \frac{\mu(H) - \mu_0}{\mu_0} \nabla \cdot \vec{H} + \frac{1}{\mu_0} \vec{H} \cdot \nabla \mu$$

$$= \frac{\mu(H) - \mu_0}{\mu_0} \nabla \cdot \vec{H} + \frac{1}{\mu_0} \frac{d\mu}{dH} \vec{H} \cdot \nabla H \tag{11}$$

In practice, however, it will probably be necessary to evaluate these derivatives numerically, which could cause problems.

Thus as an alternative to the straightforward application of (4) one might first divide the permeable medium into a number of elementary volumes V_i and assume that the permeability is a constant, μi, inside each volume. With this assumption (4) becomes

$$\phi(\vec{x}) = \phi_0(\vec{x}) + \frac{1}{4\pi} \sum_i \int_{S_i} \frac{M(\vec{x}') \cdot \vec{m}\, dS}{|\vec{x} - \vec{x}'|} \tag{12}$$

where the integrals in the sum are over the surfaces bounding the elementary volumes. From this point on the calculation could proceed as indicated above, except that the permeabilities μi would be approximated by the average value of $\mu(H(\vec{x}))$ inside the i'th volume.

C. Numerical Methods

If great accuracy is required, and it is necessary to account, in detail, for flaw geometry, and the magnetic field and stress dependence of the permeability, then the iterative solution described above will undoubtedly require the use of a large-scale digital computer. In such cases one might question the need to generate a good initial approximation to the potential; i. e., one might argue that if a large computer is needed then it is best to ignore approximate solutions and simply solve the potential problem directly by a totally numerical method. There are indeed many workers who hold this view and, as a result, there are presently available several very sophisticated computer programs for solving the nonlinear magnetostatics problem.

To illustrate the numerical approach we will describe one example, the magnetization vector program developed by Robertson and Karmaker.[26] Their starting point is the following integral equation, which is easily derived from the well-known relationship between the magnetization vector $\vec{M}$ and the vector potential:

$$\vec{M}(\vec{x}) - \frac{1-\mu_r(\vec{x})}{4\pi}\int \nabla_{\vec{x}} \times \left(\frac{\vec{M}(\vec{x}') \times (\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}\right) d^3x' = \frac{1-\mu_r(\vec{x})}{\mu_0}\vec{B}_0(\vec{x}) \tag{13}$$

where $B_o(x)$ is the flux density due to the excitation field alone, and μ_r is the relative permeability, μ/μ_o. In the computations described by Robertson and Karmaker, μ_r was assumed constant; in our discussion, however, we will allow μ_r to vary with position, as it does in the general nonlinear problem.

Next the vector operation is carried out and the vector integral equation is written in component form. The magnetized medium is then divided into N elementary volumes ΔV_j which are assumed to be small enough that the magnetization and permeability can be assumed constant inside each volume. The integral then reduces to a sum and one is left with three sets of equations for the components of M. The X-component equation, for example, is

$$M_x(i) - \frac{1-\mu_r(i)}{4\pi}\sum_{j=1}^{N}\left[\alpha_1(j)M_x(j) + \beta_1(j)M_y(j) + \beta_2(j)M_z(j)\right]\Delta V_j = \frac{1-\mu_r(i)}{\mu_0}B_{xo}(i) \tag{14}$$

where the α's and β's are functions of the coordinates defined by Robertson and Karmaker. This equation is then combined with the equations for $M_y(i)$ and $M_z(i)$ to form a 3N dimensional matrix equation which is solved by an interative method for a given choice of the permeabilities $\mu_r(i)$. If variable permeability is to be allowed one would then use this first solution for M to compute the field strength in each volume, according to

$$H = \frac{\mu_0}{\mu(H) - \mu_0} M \tag{15}$$

and from this result determine a new set of permeabilities $\mu_r(i)$. The entire process would then be repeated until successive estimates of M agree to some pre-determined accuracy. The final steps in the determination of the field would then be the calculation of that part of the vector potential due to the magnetization,

$$\vec{A}'(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{M}(\vec{x}') \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} d^3x' \tag{16}$$

followed by the calculation of the flux,

$$\vec{B}(\vec{x}) = \vec{B}_0 + \nabla \times \vec{A}'(\vec{x}) \tag{17}$$

In a sample problem run by the authors in which only 10 volume elements were considered, the computer time required to determine $\vec{M}$ in the 10 elements was about 5 seconds, including compilation time, on an IBM 360/65. However, this problem was run for constant permeability and so did not require the extra permeability interations described here. From computer time reports by other authors(27) who used similar programs, it is estimated that a nonlinear computation would require 2 to 5 minutes on a comparable computer.

Other programs now in use, primarily in magnet design applications, were recently reviewed by Colonias(27), who presented the summary shown in Table A-1. Also of interest is the program described by Hwang and Lord who used a variational principle involving the vector potential. The application of their program to the study of flaw leakage fields is reported in the following paper:

> J. H. Hwang and W. Lord, "Magnetic Leakage Field Signatures of Material Discontinuities", Proc. 10th Symposium on NDE, Southwest Research Institute (1975)

Another development of interest was reported by Halbach (referenced in Section A of this Appendix) who has extended the program POISSON, which

TABLE A-1

COMPUTER PROGRAMS FOR MAGNETIC FIELD COMPUTATIONS (Ref. 27)

Name	Method of Solution	Type of Grid	Remarks
TRIM	Finite difference Vector potential	Triangular, variable	General purpose. Good agreement with measurements. Two-dimensional simulation.
GFUN	Integral formulation Dipole magnetization	None	General purpose. Good agreement with measurements. Two- and three-dimensional simulation.
MARE	Finite difference Two potential: Scalar & Vector	Rectangular	Good agreement with measurements. Fast. Less satisfactory for highly saturated magnets.
MAFCO	Analytic (elliptic integrals)	None	Two- or three-dimensional. No ferromagnetic material allowed. Generalized boundaries.

computes two-dimensional fields by a complex potential method, to the treatment of nonlinear problems. Still other programs in use or under development are discussed in reports on the Reno Conferences on Magnetic Fields (published by the Electrical Engineering Department, University of Nevada, Reno) and in the following articles:

I. Lucas, J. Appl. Phys 47, 1645 (1976)

C. S. Holzinger, IEEE Trans. Mag. 6, 60 (1970)

D. J. Kozakoff and F. O. Simons, Jr., IEEE Trans. Mag. 6, 828 (1970)

A. M. Winslow, J. Computer Phys 1, 149 (1967)

Finally, the proceedings of a conference referenced by Colonias[27], but not yet available, can be expected to contain the latest developments in this field. The reference is

Proc. Conference on the Computation of Magnetic Fields "COMPUMAG", Rutherford Laboratory, England (1976)

APPENDIX B

A MAGNETIC PARTICLE DIFFUSION MODEL

The fundamental equation that governs the diffusion of particles in an arbitrary force field is easily obtained from the continuity equation, which is[28]

$$\frac{\partial n}{\partial t} + \nabla \cdot n\vec{u} = 0 \tag{1}$$

where

$n = n(\vec{x}, t)$ is the particle density, and
$\vec{u} = \vec{u}(x, t)$ is the average particle velocity.

According to diffusion theory[13], the particle current density, i.e., the number of particles per second passing through a unit area normal to $\vec{u}$, is given by

$$n\vec{u} = \mu n\vec{F} - D\nabla n \tag{2}$$

where

μ is the particle mobility (a constant),
$\vec{F} = \vec{F}(x, t)$ is the force on a particle at $\vec{x}$, t, and
D is the diffusion coefficient (also a constant).

Substitution of (2) in (1) gives

$$\frac{\partial n}{\partial t} - D\nabla^2 n + \mu\vec{F}\cdot\nabla n + \mu n\nabla\cdot\vec{F} = 0 \tag{3}$$

which is the time-dependent diffusion equation.

In the magnetic particle inspection problem we are concerned with the diffusion of magnetized particles under the influence of gravity, an inhomogeneous magnetic field $\vec{H}$ (x), and, possibly, other forces resulting from interparticle interactions. In any case, as all of these forces are conservative we can write

$$\vec{F}(\vec{x},t) = -\nabla U(\vec{x},t) \tag{4}$$

where $U(x, t)$ is the potential energy. If we choose the Z axis in the vertical direction, then, for particles of mass m, volume v and susceptibility χ we have

$$U(\vec{x},t) = mgz - \tfrac{1}{2}\chi v H^2(\vec{x},t) + U_p(\vec{x},t) \tag{5}$$

where $U_p(x, t)$ is the interparticle interaction energy, and g is the acceleration due to gravity.

If we let the Z = 0 plane represent the surface of the magnetized specimen, then, because particles cannot pass through this plane, the solution of (3) is subject to the condition that there be no particle current in the Z direction at Z = 0. Thus, from (2) we have the boundary condition

$$\mu M F_z \Big)_{z=0} = D \frac{\partial M}{\partial z} \Big)_{z=0} \tag{6}$$

We seek an approximate solution of the diffusion equation subject to the boundary condition (6) for an arbitrary force field of the form given by (4) and (5), in terms of an arbitrary initial distribution $M_0(x)$ which is specified at the time t = 0. In developing this solution we will assume that the potential U is independent of time. Actually, this poses no real restriction because U can always be approximated by a time-independent function for short time intervals, and, as we shall see later, the solution for a time-dependent field can therefore be generated by successive applications of the solution for a time independent field.

To generate an approximate solution we make use of the well-known fact that when the system is in thermodynamic equilibrium, the particle density is[28]

$$M(\bar{x}) = \text{constant} \times e^{-\beta U(\bar{x})} \tag{7}$$

where $\beta = 1/kT$. We therefore make the substitution

$$M(\bar{x},t) = \phi(\bar{x},t)\, e^{-\beta U(\bar{x})} \tag{8}$$

and seek a solution for the unknown function $\phi(x, t)$. The reason for making this particular substitution is that the equilibrium solution (7) suggests that if the system is near equilibrium ϕ will be a slowly varying function of x. Thus, as we shall see shortly, a first order approximate solution can be obtained by ignoring terms containing spatial derivatives of ϕ. For the present, however, we will retain such terms and derive an exact integral equation for ϕ.

Substitution of (8) in (3) gives

$$\frac{\partial \phi}{\partial t} - D\nabla^2\phi - \mu \vec{F} \cdot \nabla\phi = 0 \tag{9}$$

(In deriving this equation we have used the Einstein relation, $\mu = \beta D$, which follows easily from (2) and (7), and the fact that the current density vanishes in thermodynamic equilibrium). From (6) and (8) we have the boundary condition

$$\left.\frac{\partial \phi}{\partial z}\right)_{z=0} = 0 \tag{10}$$

and the initial condition on ϕ is

$$\phi(\vec{x}, 0) \equiv \phi_0(\vec{x}) = M_0(\vec{x}) e^{\beta U(\vec{x})} \tag{11}$$

From the mathematical theory of the conduction of heat in solids (see, for example, H. S. Carslaw and J. C. Jaeger, "Conduction of Heat in Solids," Oxford U. Press, London (1959) Chapter XIV.), we know that the solution of the equation

$$\frac{\partial \psi}{\partial t} - D\nabla^2 \psi = S(\vec{x}, t)$$

is

$$\psi(\vec{x},t) = \int G(\vec{x}, \vec{x}'; t)\, \psi_0(\vec{x}')\, d^3x'$$

$$+ \int_0^t \int G(\vec{x}, \vec{x}'; t-t')\, S(\vec{x}', t')\, d^3x'\, dt'$$

where $\qquad \psi_0(\vec{x}) = \psi(\vec{x}, t=0)$

The Green's function G satisfies the homogeneous equation

$$\left(\frac{\partial}{\partial t} - D\nabla_{\vec{x}}^2\right) G(\vec{x}, \vec{x}'; t-t') = 0 \tag{12}$$

subject to the same boundary conditions as $\psi(\vec{x}, t)$, and also has the property

$$\lim_{t' \to t} G(\vec{x}, \vec{x}'; t-t') = \delta(\vec{x} - \vec{x}') \tag{13}$$

It follows that if we can find a Green's function for the problem defined by (9), (10) and (11), that the differential equation (9) can then be replaced by the integral equation

$$\phi(\vec{x}, t) = \int G(\vec{x}, \vec{x}'; t)\, \phi_0(\vec{x}')\, d^3x'$$
$$+ \mu \int_0^t \int G(\vec{x}, \vec{x}'; t-t')\, \vec{F}(\vec{x}') \cdot \nabla_{\vec{x}'} \phi(\vec{x}', t')\, d^3x'\, dt' \tag{14}$$

A Green's function that satisfies (12) and (13) is[13]

$$\tilde{G}(\vec{x},\vec{x}\,';\tau) = \frac{e^{-\frac{|\vec{x}-\vec{x}\,'|}{4D\tau}}}{(4\pi D\tau)^{3/2}}$$

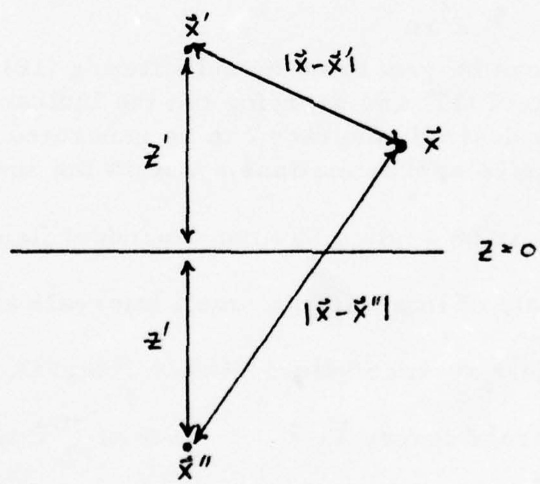
However, as this function does not satisfy the condition

$$\left.\frac{\partial G(\vec{x},\vec{x}\,';\tau)}{\partial z}\right|_{z=0} = 0 \tag{15}$$

we try instead the function

$$G(\vec{x},\vec{x}\,';\tau) = [4\pi D\tau]^{-3/2}\left[e^{-\frac{|\vec{x}-\vec{x}\,'|^2}{4D\tau}} + e^{-\frac{|\vec{x}-\vec{x}\,''|^2}{4D\tau}}\right] \tag{16}$$

where x'' is the image point shown in the sketch below



Now the condition (15) is satisfied but instead of (13) we have

$$\lim_{t'\to t} G(\vec{x},\vec{x}\,';t-t') = \delta(\vec{x}-\vec{x}\,') + \delta(\vec{x}-\vec{x}\,'')$$

However, the second delta function does not effect the result if we limit the volume integrals in (14) to the region above the surface of the specimen (i.e., $z'>0$). Thus, with G given by (16) the integral equation for ϕ is

$$\phi(\vec{x},t) = \int_{z'>0} G(\vec{x},\vec{x}';t)\,\phi_0(\vec{x}')\,d^3x'$$

$$+\mu\int_0^t\int_{z'>0} G(\vec{x},\vec{x}';t-t')\,\vec{F}(\vec{x}')\cdot\nabla_{\vec{x}'}\phi(\vec{x}',t')\,d^3x'dt' \qquad (17)$$

To obtain an approximate solution to (17) we assume that the system is near equilibrium and that $\phi(\vec{x}, t)$ is therefore a slowly varying function of position. Thus

$$\nabla_{\vec{x}}\phi \sim 0$$

and

$$\phi(\vec{x},t) \sim \int_{z'>0} G(\vec{x},\vec{x}';t)\,\phi_0(\vec{x}')\,d^3x' \qquad (18)$$

A better approximation can be generated by substituting (18) in the integral on the right side of (17) and carrying out the indicated integrations. In fact, a solution of any desired accuracy can be generated by repeating the process until successive approximations agree to the specified accuracy.

This formalism can be applied to time dependent fields as follows:

First divide the time scale of interest into small intervals and replace $F(\vec{x}, t)$ and $U(\vec{x}, t)$ by their average values in each interval. Thus, for $t_i \le t \le t_{i+1}$ use the average force, $\vec{F}_i(\vec{x})$, in place of $\vec{F}(\vec{x}, t)$. Then, starting with the earliest time interval $0 \le t \le t_1$, calculate $\phi_0(\vec{x}') = M(\vec{x},0)\,e^{\beta U_0(\vec{x})}$ where $U_0(x)$ is the average potential in this time interval. Next apply (18) (or a higher order iterated approximation) to determine $\phi(x, t_1)$ and thus $M(\vec{x},t_1) = \phi(\vec{x},t_1)e^{-\beta U_0(\vec{x})}$

To proceed to the next time interval use

$$\phi_0(\vec{x}) = M(\vec{x},t_1)\,e^{\beta U_1(\vec{x})} = \phi(\vec{x},t_1)\,e^{\beta(U_1(\vec{x})-U_0(\vec{x}))}$$

and apply (18) in the form

$$\Phi(\vec{x},t) \sim \int_{z'>0} G(\vec{x},\vec{x}';t-t_1)\Phi_0(\vec{x}')d^3x'$$

to determine Φ in the interval $t_1 \leq t \leq t_2$. This stepwise method of calculation can then be repeated as often as necessary to cover all times of interest.

Although the formalism outlined above is quite general, in that it can be applied to particle diffusion in any conservative force field, a complication arises when interparticle interactions are introduced. This is because the potential energy function $U(\vec{x})$ then depends on the density $\mathcal{M}(\vec{x})$ and the diffusion equation becomes nonlinear. The usual approach to the solution of such problems is by iteration. Thus, one first uses some approximate function $\tilde{\mathcal{M}}_0(\vec{x})$ to determine an approximate potential $\tilde{U}_0(\vec{x})$. This potential is then used to calculate a new density $\tilde{\mathcal{M}}_1(\vec{x})$ which is used to determine a corrected potential $\tilde{U}_1(\vec{x})$, and the process is repeated until the difference between successive approximations to $\mathcal{M}(\vec{x})$ is less than some predetermined allowable error. This type of calculation is however, quite lengthy and requires considerable computer memory capacity. It is, therefore, advisable to seek an approximate method for treating interparticle interaction energies. Such an investigation lies beyond the scope of the present project but should be included as part of Task 2 in the follow-on program.

APPENDIX C

EXPERIMENTAL TASKS FOR AN INVESTIGATION OF FLUX DENSITY DETERMINATIONS

by
William L. Rollwitz
Institute Scientist

It has been determined that experiments will be needed to provide test data and proof data for the development of a mathematical model pertaining to magnetic particle inspection. The particular experimental tasks described in this Appendix are those pertaining to the simulation of flaw leakage fields, the measurement of leakage fields, and the determination of magnetic particle characteristics.

A. Leakage Field Simulation

1. Magnetic Recording Methods

The problems[(1)*] encountered in the use of magnetic wire or tape are in: (1) the process of magnetizing an extremely small volume of a thin material which has essentially the properties of a permanent magnet, and (2) the process of detecting the remanent flux in the thin magnetic material. The magnetizing process is usually confined to a very small area of the magnetic material through the use of a magnetizing head with a very small gap. A sketch is given in Figure C-1 of a magnetizing head held against a magnetic recording medium. The current in the coil produces a magnetic field in the medium much like circles centered in the gap. The graph in Figure C-2 is of the field from the gap in the magnetic material as a function of distance from the center of the gap. The high permeability of the core material restricts the field in the material so that the longitudinal component of the field (curve A in Figure C-2) is down by around 50% at the gap edge and down by 96% at one gap width of distance from the center of the gap. The vertical component of the field (curve A^1 in Figure C-2) peaks at the edge of the gap and then falls off very rapidly. As the magnetic medium moves past the gap, it will first experience the magnetic field from the gap. The magnetic field from the gap will cause an induction B in the magnetic medium according to the values of its B/H loop. When the medium is moved so that the region previously in the gap is now out of the magnetizing field, then the magnetic induction, in the part of the medium moved, is reduced to the remanence value. This process is diagrammed in Figure C-3. As the signal or voltage applied to the coil goes from zero to a, the induction in the medium, B, moves along the B/H loop (0 to A) in Figure C-3. When the signal voltage moves from a to zero, the induction in the material moves from a to a^1 in Figure C-3, leaving the remanent value of induction

*References for this Appendix are located on page 67.

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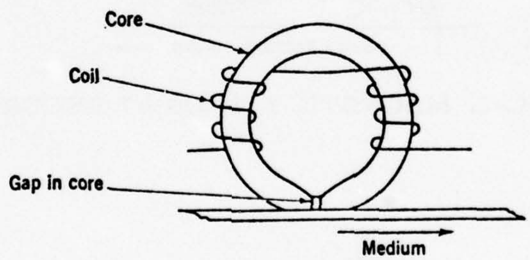

FIGURE C-1. MAGNETIZING SYSTEM FOR A MAGNETIC MEDIUM

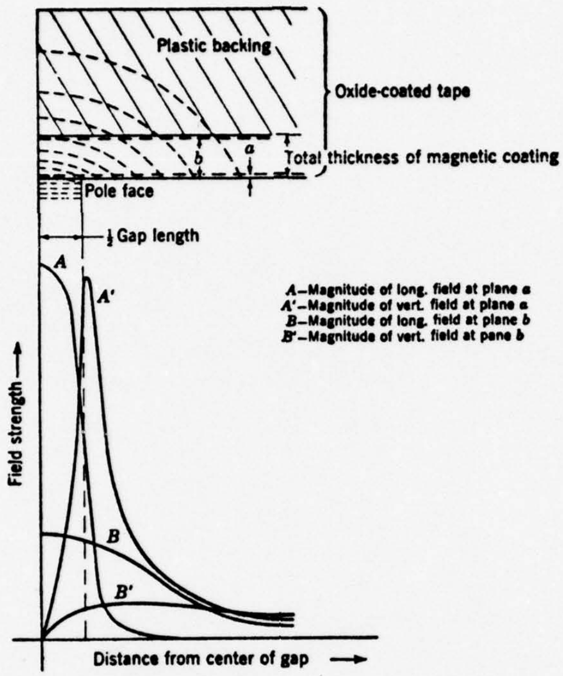

FIGURE C-2. MAGNETIC FIELDS AT RECORDING POLE

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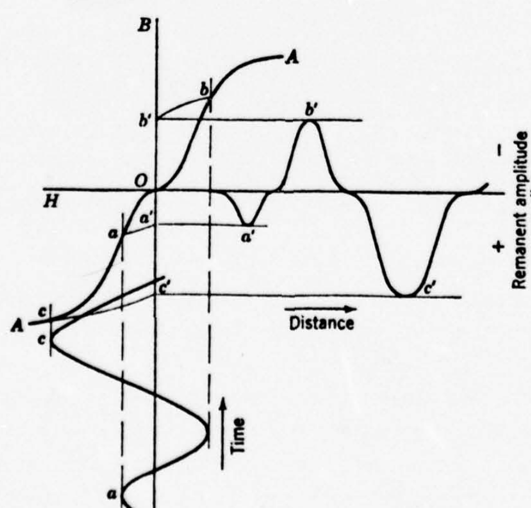

FIGURE C-3. TRANSFER CHARACTERISTIC -- ZERO-BIAS RECORDING on the medium. Thus, the signal a, b, c in Figure C-3 will produce a remanent magnetization a^1, b^1, c^1, in the magnetic medium. There is a demagnetizing[2] factor because the magnetized sections of the magnetic medium are small. If the thickness of the magnetic medium is 80% of the gap width, then the demagnetizing factor is 0.5. This means that the residual field in the magnetic medium is reduced by one-half from the remanent value.

The graphs in Figure C-2 of the field intensity around a gap show that, as one scans across the gap of a magnetic recording head, the magnitude of longitudinal component, A, of the field will be very small one gap length away from the center of the gap, reach a maximum at the center of the gap and then decrease to a very small value one gap length beyond the center of the gap. The vertical component, A^1, will be negative on the left side in Figure C-2, zero at the center of the gap, and positive to the right side of center. The negative and positive peaks of the vertical component occur just before and slightly beyond the edges of the gap (see Figure C-2). These behaviors of the longitudinal and vertical components of the fields from a magnetic recording head are similar to the behaviors of the fields from a defect when a magnetic field is applied. Therefore, a magnetic recording head can be used to produce a magnetic field which has longitudinal and vertical components similar to those from defects.

The shapes of the cross-section through the magnetic fields from a magnetic recording head are given in the graph in Figure C-4. The solid lines are contours of equal magnitude of the longitudinal field and the dashed lines are contours for equal magnitude of the vertical field. The horizontal variable is the distance from the center along the x axis normalized by the gap width. The vertical variable is the distance along the y axis normalized by the gap width. As an example, a magnetic recording head with a 0.5 mil gap can produce a field of 750 oersteds at a distance of 0.5 mils from the head with around 1.5 ampere-turns[3]. The graphs of the vertical magnetic fields around a magnetic recording head[4] are given in Figure C-5. The gap in this case is 4 milli-inches and the peak amplitude of the vertical field component is 300 oersteds.

As described previously, the magnetic recording medium has a remnant magnetic field in the magnetic material after it passes over an energized magnetic recording head. The graphs of the magnitudes of the vertical component of the remanent magnetic fields in a piece of magnetic recording tape[5] as a function of width (y direction) and length (x direction), are given in Figure C-6. The signal recorded was a series of pulses called "ones" in the non-return to zero[6] mode (NRZI). The fields are alternately positive and negative with a peak magnitude of 200 oersteds.

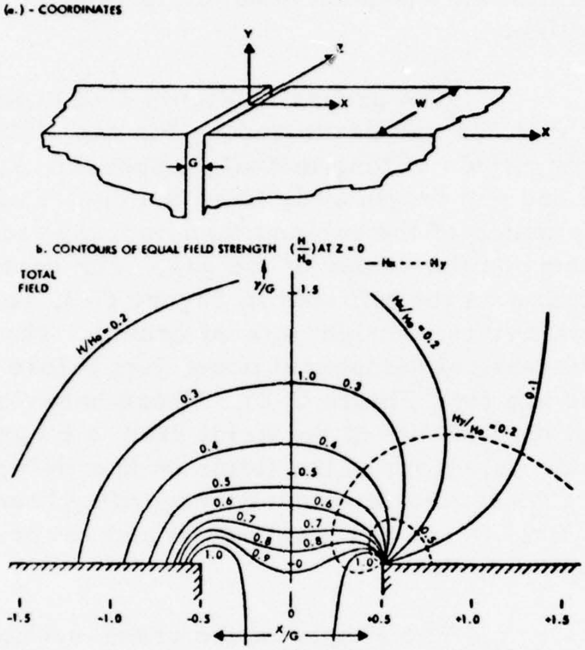

Figure 2.10. Cross section through magnetic fields near to the gap of a recording head.

(a) Coordinates.

(b) Contours of constant field strength (H/H_0) at $Z = 0$.

H = total field
H_x = horizontal component—solid line
H_y = vertical component—dashed line

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FIGURE C-4. CROSS SECTION THROUGH MAGNETIC FIELDS NEAR TO THE GAP OF A RECORDING HEAD

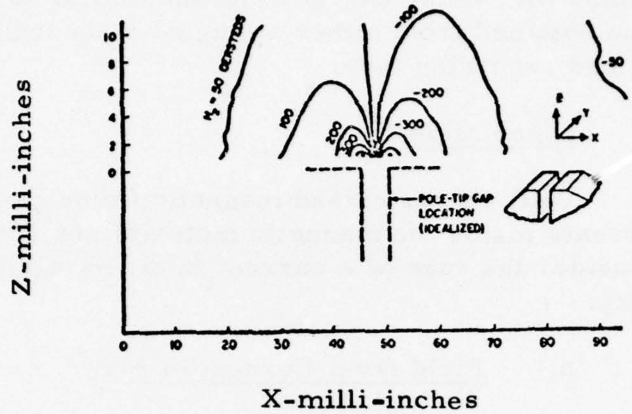

FIGURE C-5. MAP OF HEAD FIELD -- PERPENDICULAR TO GAP

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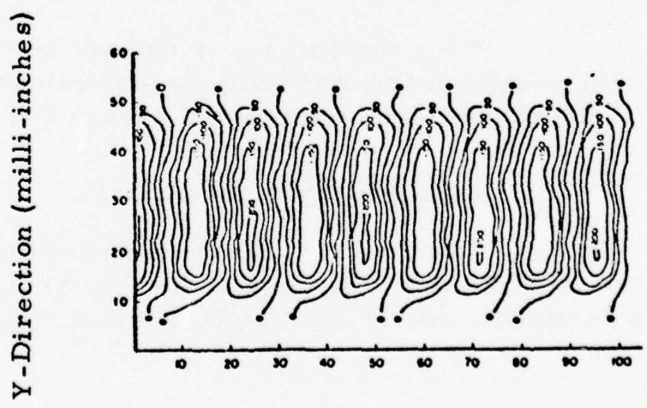

FIGURE C-6. MAP OF DRUM FIELD -- SERIES OF "ONES" -- NRZI

From the foregoing discussion, it can be concluded that magnetic fields with shape characteristics similar to those produced by defects can be obtained from either a magnetic recording head or a piece of magnetized recording tape.

2. Other Methods

Well characterized magnetic fields can also be produced by currents inside the magnetic material and outside of it. We will first consider the case of a current in air parallel to a plane air-iron boundary.

a. Field from Current in Air

A current is applied in air a distance h from the air-iron plane boundary as shown in Figure C-7. The permeability of the iron is 19 times that of the air. The method of images[7] has been used to calculate the image currents i_2 and i_1 in Figure C-7. From the image currents and the boundary conditions ($H_{1T} = H_{2T}$ and $B_{1N} = B_{2N}$) the image current values can be obtained. These currents are calculated from

$$i_2 = \frac{i(\mu_2 - \mu_1)}{(\mu_2 + \mu_1)} = -i_1 \qquad (1)$$

where the currents are as described in Figure C-7, $i_2 = -i_1 = 0.9i$.

When the current values are inserted into the field equations, the graphs of flux and equipotential values can be calculated. The graphs of the fields are given in Figure C-8.

b. Field from Current in Iron

When the current is applied through the iron, the conditions are as in Figure C-9. The applied current, i, is in the iron, the image current in the air is i, and the image current in the iron is i_2. In this case

$$i_2 = \frac{i\,\mu_2 - \mu_1}{(\mu_2 + \mu_1)} = -i_1 \qquad (2)$$

When the permeability of the iron is 19 times that of the air, then $i_2 = 0.9\,i$ and and $i_1 = -0.9i$.

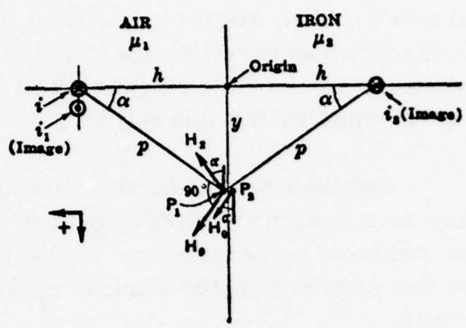

FIGURE C-7. FIELD DIRECTIONS CAUSED BY A CURRENT (i) IN AIR NEAR A PLANE IRON SURFACE

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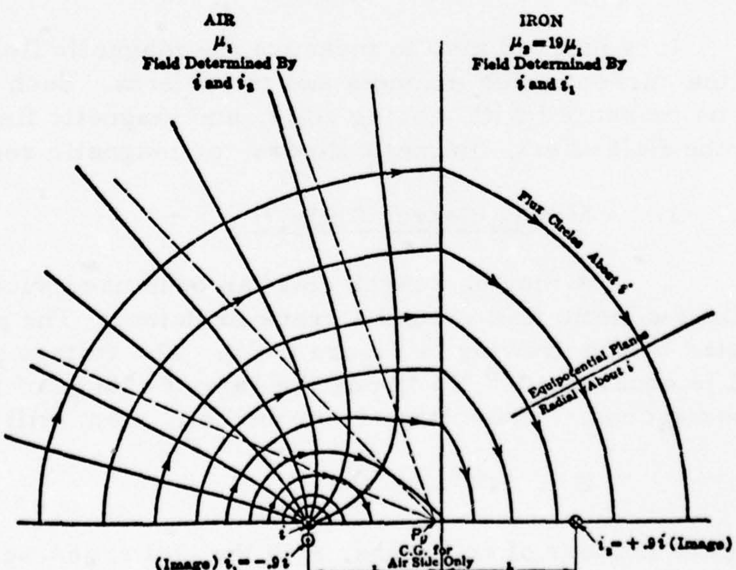

FIGURE C-8. GRAPHS OF FLUX AND EQUIPOTENTIAL LINES FROM A CURRENT (i) IN AIR NEAR A PLANE IRON SURFACE

The magnetic fields in the iron and in the air caused by these three currents are graphed in Figure C-10. The flux lines in the air are circles about the point of the supplied current i. The equipotential lines in the air are radial lines through i. Therefore, the fields in the air above an iron surface, caused by a current in the iron, are shaped like circles centered at the current. This result should be readily applicable to the determination of an analytical field whose strength is proportional to the current.

Another graph of the flux lines from a line current in air but close to a magnetic medium is given in Figure C-11[8] for the case where the relative permeability of the iron is 9. The lines of flux in the air from the current in the iron are clearly circles centered about the applied current.

In order to supply these currents, no conduction is permitted in the magnetic material. This condition can be obtained through the use of thin insulation around the wire. This slight gap will need to be included in the calculation if the thickness is an appreciable part of the distance h.

B. Measuring Magnetic Fields

It is desired also to measure the magnetic fields from all sources: the currents, the magnets and the defects. Such magnetic fields can be measured with moving coils, and magnetic field transducers which use the Hall effect, magnetic diodes, or magnetic resistors.

1. Moving Search Coils[9]

A moving search coil has been used successfully to measure the magnetic flux around a crack or defect. The principle is demonstrated by the drawing in Figure C-13. The voltage picked up by the coil is equal to 10^{-8} NA times the rate of change of the induction in gauss per second. The voltage from the coil, then, will be

$$E = 10^{-8} NA \frac{dB}{dt} \tag{3}$$

where N is the number of coil turns, A is the coil cross-sectional area and B is the intensity of the longitudinal flux density or induction. Because of the direction of the coil axis and the motion, the voltage generated is proportional to the rate of change of flux in the longitudinal direction or parallel to the surface of the coil. The tangential flux component does not induce a voltage in the coil for the coil path shown in Figure C-13. Since only the change of longitudinal field (call that the x direction) couples to the coil, then Equation (3) can be written as

At point P_1 (Figure C-9), use i and i_2 in μ_1

$$H_{1r} = -\frac{(i + i_2)}{2\pi p}\cos\alpha = -\frac{h}{2\pi p^2}(i + i_2)$$

$$B_{1N} = +\mu_1\frac{(i + i_2)}{2\pi p}\sin\alpha = +\frac{y\mu_1}{2\pi p^2}(i + i_2)$$

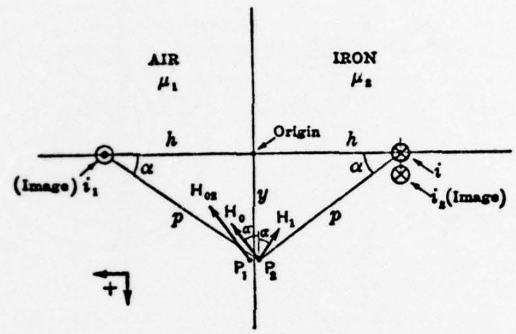

FIGURE C-9. FIELD DIRECTIONS CAUSED BY A CURRENT (i) IN IRON NEAR A PLANE AIR SURFACE

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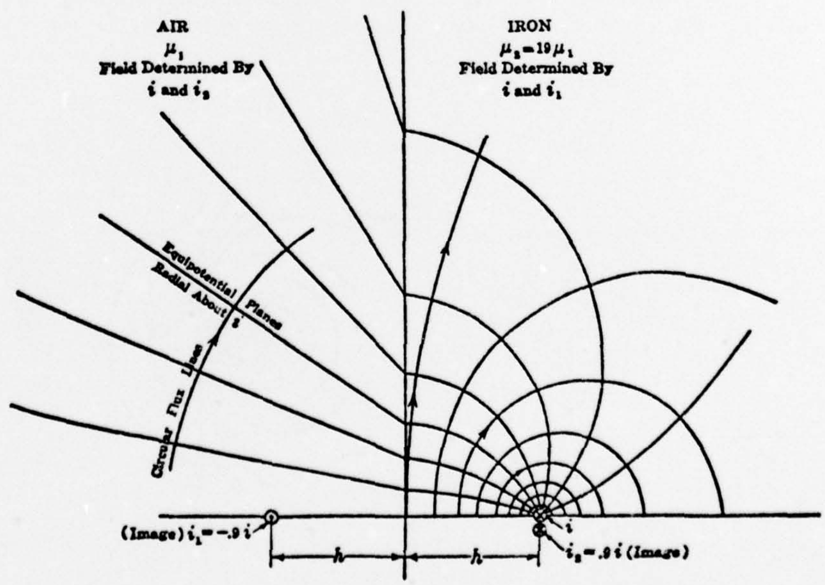

FIGURE C-10. CURRENT (i) IN IRON NEAR A PLANE AIR SURFACE

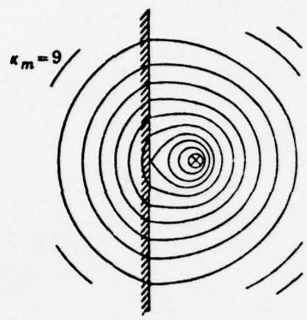

FIGURE C-11. MAGNETIC FLUX LINES FROM A LINE CURRENT IN FRONT OF A MAGNETIC MEDIUM

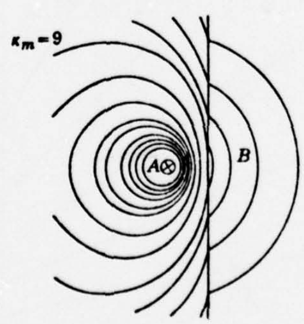

FIGURE C-12. MAGNETIC FLUX LINES FROM A LINE CURRENT INSIDE A MAGNETIC MEDIUM

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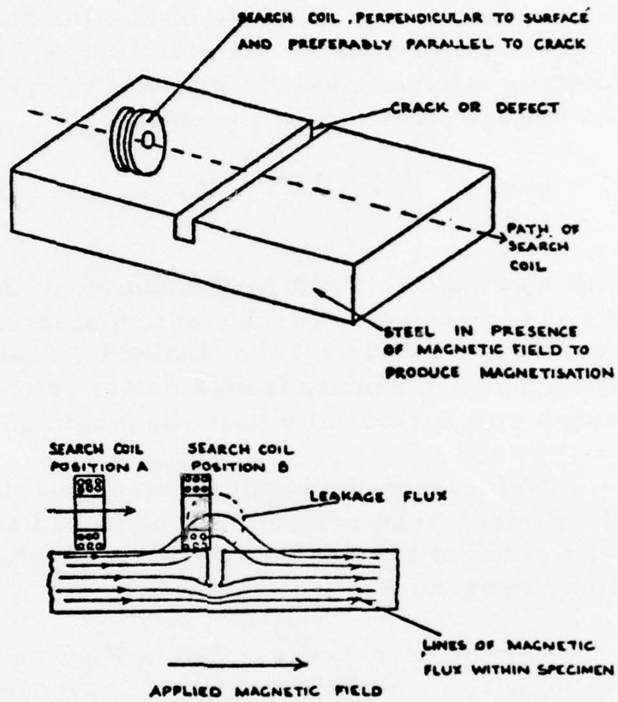

FIGURE C-13. THE PRINCIPLE OF FLUX LEAKAGE MEASUREMENT USING A SEARCH COIL

$$E = 10^{-8} \, NA \, \frac{dB}{dx} \, \frac{dx}{dt} \qquad (4)$$

The rate of change of B with x is the slope of the flux curve of Figure C-14. The rate of change of distance (x) with time is the velocity of the coil. If the velocity is constant over the distance wherein the flux density changes, then the voltage picked up by the coil is

$$E = 10^{-8} \, NA v \frac{dB}{dx} \qquad (5)$$

The voltage picked up by the coil will be that shown as the search coil output in Figure C-14. When the search coil output is integrated, the flux density curve of Figure C-14 will be obtained. Therefore, the curve of the longitudinal flux density from a defect or current could be obtained by moving a coil horizontally over the length of the field.

To measure the vertical component of the flux density, the coil will need to be rotated 90° and moved across the defect. The actual graph of the flux density versus distance will again be obtained by integration.

For the case[9] described in Figures C-13 and C-14, the search coil used had an outside diameter of 3mm, and a width of 1.5 mm. The coil was wound with 1000 turns of wire 0.7 milli-inches in diameter. The coil was passed over the defect at a rate of 3 meters per second.

To facilitate these measurements, the coil could be vibrated at one position to determine the slope of the flux density versus distance at one point. The total curve could then be constructed from the point-by-point data.

It may also be practical to attach the coil to a spring and trigger mechanism so that the coil is flipped through the flux density curve, recorded and processed to give the desired flux versus distance curve.

2. Fixed Search Coils

In the case of fields in magnetic tape, measurements can be made by moving the tape past a fixed coil. One report[10] of such a measurement used a single turn coil plated on a glass slide. The single turn was 1.16 inch square and 1/64 inch thick. The flux density measured had a peak value of around 30 gauss. The recorded signal was a sine wave of 1600 Hz and the tape speed was 30 inches/second.

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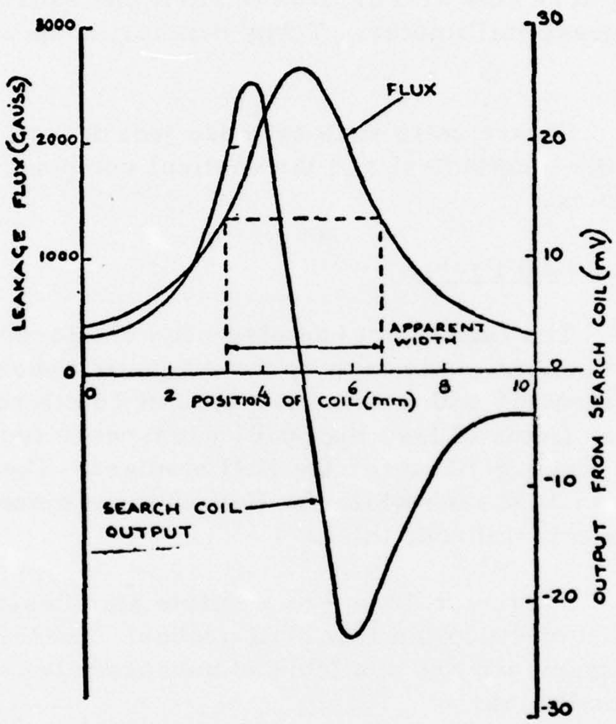

FIGURE C-14. OUTPUT VOLTAGE FROM SEARCH COIL AND LEAKAGE-FLUX AS FUNCTIONS OF POSITION

3. Search Coil Construction

For some time now, the NDE section of Southwest Research Institute has been constructing very small coils to detect the magnetic fields from defects. For this work, the test specimen is usually moved or rotated under the stationary coil although there have been cases where the coil moved. Because of these experiences, there are personnel with many years experience in winding very small coils of 0.5 milli-inch diameter wire. It is possible and practical to construct a multi-turn coil with an area of from one square millimeter to one-quarter square millimeter. Turns numbering up to 100 may be practical.

Square coils such as those just described can be used to measure both the longitudinal and the vertical components of the leakage fields around defects.

4. Hall Probes

The Hall effect has also been employed(4) to measure the fields placed upon magnetic tape by the magnetic recording process. Probes were constructed with a sensitive area of 10 microns by 10 microns. With these probes, fields of less than 0.01 gauss were found to be measurable using evaporated bismuth films for the Hall element. The probes had an overall thickness of 1/32 inch while the Hall element plated on the substrate was only two to twenty microns thick.

Southwest Research Institute also has experience with the fabrication of deposited film Hall probes. Therefore, the expertise and the experience are available to successfully construct Hall probes of very small size.

C. Measuring the Properties of Magnetic Powders

There are particles with such wide and varied properties that it will be necessary to determine the particle size distribution, the particle shape distribution, the agglomeration properties, and the magnetic properties. For the magnetic properties it would be useful to know the distribution of the magnetic moments, the magnetic anisotropy field and the B/H curve. These needs will be discussed in the following paragraphs.

1. Measuring Size and Shape Distribution

The size distribution can be determined by weighing the amounts remaining in a series of sieves. The number of particles in each size range can then be counted. An estimate can then be made of the number in each range which have shapes with a length-to-diameter ratio of one to six. With these pieces of information, both the size distribution and the shape distribution can be determined.

2. Wet Materials Concentrations

The ability or lack of ability for the particles to agglomerate, whether dry or in a liquid, depends upon the concentration and the magnetic properties of the particles. The concentration of particles in the wet bath materials for magnetic particle inspection can be determined through a settling test in a centrifuge. The usual concentration for the red and black particles is 1.5 to 2 cm^3 of particles per 100 cm^3 total volume or from 1.5% to 2% volume per volume. For the fluorescent materials, the volume concentration of particles is between 0.2 and 0.4 cm^3 per 100 cm^3.

3. Magnetic Properties

The Cotton-Mouton effect has been used[11] to determine some of the magnetic properties of particles in solution. The Cotton-Mouton effect is a change in the induced optical birefringence of the liquid containing the particles caused by the orientation of the particles in AC and DC magnetic fields. An analysis of the field dependence yields the following:

- the average magnetic dipole moment
- the width of the distribution of the dipole moment
- the magnetic anisotropy field
- the distribution of switching fields

To perform the measurements, the sample is inserted into the optical path as shown in Figure C-15 and into the field of the magnet M to be used for magnetizing. The light beam from the source, L, passes through a polarizing prism, P, the sample S, and a second polarizer A, to the detector photocell, C. The intensity of the light is measured as a function of the applied field when the magnetic field is generated from both direct and alternating current sources.

When the particle concentration is made low enough, the magnetic interactions between the particles is low. In a magnetic field which orients the particles, the suspension becomes birefringent. The measurement procedures are given in the referenced article and need not be repeated here. It appears, from the data presented in the paper, that high accuracy will be difficult to obtain because a value of 2×10^5 A/m was theoretically expected while values of between 0.8×10^5 and 1.5×10^5 A/m were measured.

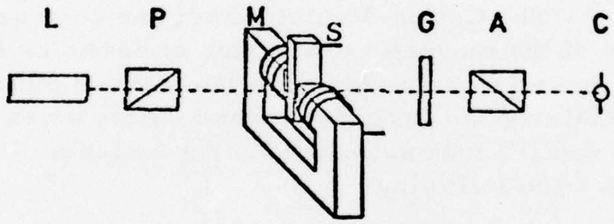

FIGURE C-15. EXPERIMENTAL SET-UP FOR THE COTTON-MOUTON EFFECT

4. Hysteresis Loops

The hysteresis loop gives the saturation magnetization, the retentivity and the coercive force. A successful magnetic loop tracer is described in Reference (12), page 249. The drawing of the coil arrangement used is given in Figure C-16. There is an exciting coil around the sample holding tube. There are also six coils around the exciting coil which are mutual inductors that serve to: (1) measure the peak current and hence, the peak magnetizing force in the exciting coil, (2) balance the air flux in the pickup coil, and (3) calibrate the vertical scale of the oscilloscope in kilogauss. When properly balanced and calibrated, hysteresis loops can be obtained from wet or dry magnetic powders inserted into the sample volume.

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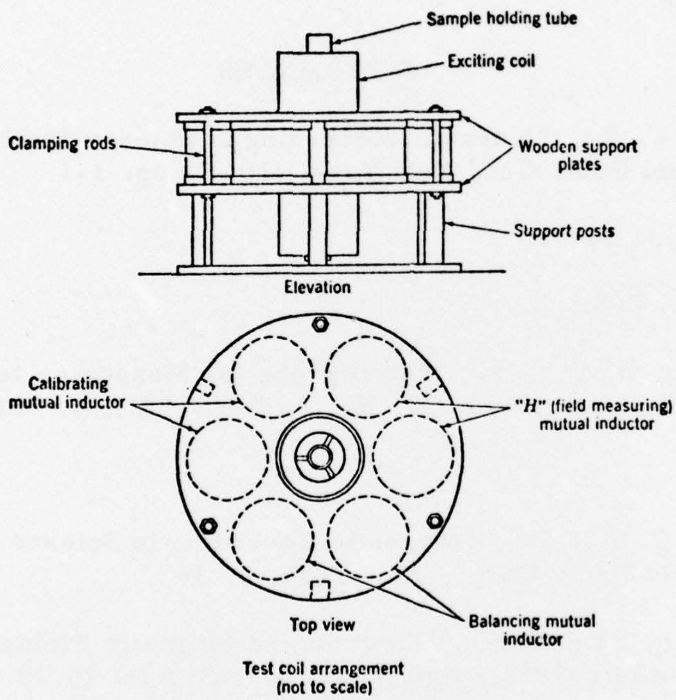

FIGURE C-16. COIL ARRANGEMENT USED TO OBTAIN THE HYSTERESIS CURVES OF SAMPLES OF MAGNETIC PARTICLES BOTH DRY AND WET

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